

**Generalized evolution of linear bias: A tool to test gravity**

S. Basilakos\*

*Academy of Athens, Research Center for Astronomy and Applied Mathematics, Soranou Efessiou 4, 11527, Athens, Greece, and High Energy Physics Group, Department ECM, University de Barcelona, Av. Diagonal 647, E-08028 Barcelona, Spain*

M. Plionis†

*Institute of Astronomy and Astrophysics, National Observatory of Athens, Thessio 11810, Athens, Greece, and Instituto Nacional de Astrofísica, Óptica y Electrónica, 72000 Puebla, Mexico*

A. Pouri

*Faculty of Physics, Department of Astrophysics—Astronomy—Mechanics University of Athens, Panepistemiopolis, Athens 157 83, and Academy of Athens, Research Center for Astronomy and Applied Mathematics, Soranou Efessiou 4, 11527, Athens, Greece*

(Received 30 March 2011; published 23 June 2011)

We derive an exact analytical solution for the redshift evolution of linear and scale-independent bias, by solving a second-order differential equation based on linear perturbation theory. This bias evolution model is applicable to all different types of dark energy and modified gravity models. We propose that the combination of the current bias evolution model with data on the bias of extragalactic mass tracers could provide an efficient way to discriminate between geometrical dark energy models and dark energy models that adhere to general relativity.

DOI: [10.1103/PhysRevD.83.123525](https://doi.org/10.1103/PhysRevD.83.123525)

PACS numbers: 98.80.-k, 95.35.+d, 98.65.Dx, 98.80.Bp

**I. INTRODUCTION**

It is well known that the large-scale clustering pattern of different extragalactic mass tracers (galaxies, clusters, etc.) trace the underlying dark matter distribution in a biased manner [1,2]. Such a biasing is assumed to be statistical in nature; with galaxies and clusters being identified as high peaks of an underlying, initially Gaussian, random density field. The linear and scale-independent bias factor,  $b$ , is thus defined as the ratio of the mass-tracer overdensity to that of the underlying mass overdensity, or equivalently as the ratio of the square root of the mass-tracer 2-point correlation function to that of the underlying mass correlation function. Furthermore, the redshift evolution of bias,  $b(z)$ , is very important in order to relate observations with models of structure formation and has been shown to be a monotonically increasing function of redshift.

There are two basic families of analytic bias evolution models. The first, called the *galaxy merging* bias model, utilizes the halo mass function and is based on the Press-Schechter [3] formalism, the peak-background split [2], and the spherical collapse model [4]. Many studies have compared the prediction of the *merging* bias model with numerical simulations, and beyond an overall good agreement, differences have been found in the details of the halo bias. These differences have led to modifications of the original model to include the effects of ellipsoidal collapse [5] and to either provide new fitting bias model parameters [6], or new forms of the bias model fitting function [7]

or even a non-Markovian extension of the excursion set theory [8].

The second family of bias evolution models assumes a continuous mass-tracer fluctuation field, proportional to that of the underlying mass, and the tracers act as “test particles.” In this context, the hydrodynamic equations of motion and linear perturbation theory are used. This family of models can be divided into two subfamilies:

- (a) The so-called *galaxy-conserving* bias model uses the continuity equation and the assumption that tracers and underlying mass share the same velocity field [9–12]. Then the bias evolution is given as the solution of a first order differential equation, and Tegmark and Peebles [11] derived:  $b(z) = 1 + (b_0 - 1)/D(z)$ , with  $b_0$  is the bias factor at the present time and  $D(z)$  the growing mode of density perturbations. However, this bias model suffers from two fundamental problems: *the unbiased problem* i.e., the fact that an unbiased set of tracers at the current epoch remains always unbiased in the past, and *the low redshift problem* i.e., the fact that this model represents correctly the bias evolution only at relatively low redshifts  $z \lesssim 0.5$  [13]. Note that the authors of [14] have extended this model to also include an evolving mass-tracer population in a  $\Lambda$ CDM cosmology.
- (b) A model which is based on the basic equation for the evolution of linear density perturbations, and on the assumption of linear and scale-independent bias, which are used to derive a second-order differential equation for the bias, the approximate solution of which provides the evolution of bias (see [15,16]). The provided solution applies to cosmological

\*svasil@academyofathens.gr

†mplionis@astro.noa.gr

models, within the framework of general relativity, with a constant in time dark energy equation of state parameter (i.e., quintessence or phantom).

In this article, we extend the original Basilakos and Plionis [15] bias evolution model to provide an exact solution valid for all dark energy and modified gravity cosmologies. This implies that the current bias evolution model can be used to put constraints on dark energy models as well as to investigate possible departures from general relativity.

## II. THE EVOLUTION OF THE LINEAR GROWTH FACTOR

In this section, we discuss the basic equation which governs the behavior of the matter perturbations on sub-horizon scales and within the framework of any dark energy (hereafter DE) model, including those of modified gravity (“geometrical dark energy”). For these cases, a full analytical description can be introduced by considering an extended Poisson equation together with the Euler and continuity equations. Consequently, the evolution equation of the matter fluctuations, for models where the DE fluid has a vanishing anisotropic stress and the matter fluid is not coupled to other matter species (see [17–23]), is given by

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\text{eff}}\rho_m\delta_m = 0, \quad (2.1)$$

where  $\rho_m$  is the matter density and  $G_{\text{eff}}(t) = G_N Y(t)$ , with  $G_N$  denoting Newton’s gravitational constant.

For those cosmological models which adhere to general relativity, [ $Y(t) = 1$ ,  $G_{\text{eff}} = G_N$ ], the above equation reduces to the usual time evolution equation for the mass density contrast [24], while in the case of modified gravity models (see [17,21–23]), we have  $G_{\text{eff}} \neq G_N$  [or  $Y(t) \neq 1$ ]. In this context,  $\delta_m(t) \propto D(t)$ , where  $D(t)$  is the linear growing mode (usually scaled to unity at the present time). Changing variables from  $t$  to  $a$ , Eq. (2.1) becomes

$$\frac{d^2\delta_m}{da^2} + A(a)\frac{d\delta_m}{da} - B(a)\delta_m = 0, \quad (2.2)$$

where

$$A(a) = \frac{d \ln E}{da} + \frac{3}{a} \quad \text{and} \quad B(a) = \frac{3\Omega_m}{2a^5 E^2(a)} Y(a) \quad (2.3)$$

with  $\Omega_m$  being the density parameter at the present time and  $E(a) = H(a)/H_0$  is the normalized Hubble function.

Useful expressions of the growth factor have been given by [24] for the  $\Lambda$ CDM cosmology. Several works have also derived the growth factor for  $w(z) = \text{const}$  DE models (see [25–27]), and for the braneworld cosmology [17]. Also Linder and Cahn [21] derived similar expressions for geometrical dark energy models in which the Ricci scalar varies with time, as well as for models with a time-varying equation of state, while for the scalar tensor and  $f(R)$

models the growth factors are provided by Gannouji *et al.* [23] and Tsujikawa *et al.* [22].

## III. THE GENERAL EVOLUTION OF BIAS

In Basilakos and Plionis [15], we assumed that for the evolution of the linear bias, the effects of nonlinear gravity and hydrodynamics (merging, feedback mechanisms, etc.) can be ignored (see [10,11]). Then, using linear perturbation theory in the context of general relativity [ $Y(t) = 1$ ,  $G_{\text{eff}} = G_N$ ] we obtained a second-order differential equation that describes the evolution of the linear bias factor,  $b$ , between the background matter and the mass-tracer fluctuation field:

$$\ddot{y}\delta_m + 2(\dot{\delta}_m + H\delta_m)\dot{y} + 4\pi G_{\text{eff}}\rho_m\delta_m y = 0, \quad (3.1)$$

where  $y = b - 1$ . Below, we will prove that the above expression is valid for any cosmological model<sup>1</sup> including those of modified gravity, with  $G_{\text{eff}} = G_N Y(t)$ . Since we also make the same assumption, as in our original formulation, that the tracers and the underlying mass distribution share the same velocity field and thus the same gravity field, the above equation is valid also for cosmological models with a modified theory of gravity. Using the latter we have

$$\dot{\delta}_m + \nabla u \simeq 0 \quad \text{and} \quad \dot{\delta}_{\text{tr}} + \nabla u \simeq 0, \quad (3.2)$$

from which we obtain

$$\dot{\delta}_m - \dot{\delta}_{\text{tr}} = 0. \quad (3.3)$$

Now since we assume linear biasing, we have  $\delta_{\text{tr}} = b\delta_m$ , and using  $y = b - 1$ , we get that  $d(y\delta_m)/dt = 0$ . Differentiating the latter twice, we then get  $\ddot{y}\delta_m + 2\dot{y}\dot{\delta}_m + y\ddot{\delta}_m = 0$ . Solving for  $y\ddot{\delta}_m$ , using the fact that  $y\dot{\delta}_m = -\dot{y}\delta_m$  and Eq. (2.1) we finally obtain Eq. (3.1).

Transforming Eq. (3.1) from  $t$  to  $a$ , we simply derive the evolution equation of the function  $y(a)$  [where  $y(a) = b(a) - 1$ ], which has some similarity with the form of Eq. (2.2) as expected. Indeed this is

$$\frac{d^2 y}{da^2} + \left[ A(a) + \frac{2f(a)}{a} \right] \frac{dy}{da} + B(a)y = 0, \quad (3.4)$$

where  $f(a)$  is the growth rate of clustering, a parametrization of the linear matter perturbations, given by

$$f(a) = \frac{d \ln \delta_m}{d \ln a} = \frac{d \ln D}{d \ln a} = \Omega_m^\gamma(a), \quad (3.5)$$

where  $\Omega_m(a) = \Omega_m a^{-3}/E^2(a)$  and  $\gamma$  is the growth index, originally introduced by Wang and Steinhardt [26].

<sup>1</sup>The current theoretical approach does not treat the possibility of having interactions in the dark sector. Also discussions beyond the linear biasing regime can be found in [28] (and references therein).

Integrating Eq. (3.5) we obtain the growth factor for any type of dark energy:

$$D(a) = ae^{\int_0^a (dx/x)[\Omega_m^{\gamma(x)} - 1]}. \quad (3.6)$$

In Basilakos and Plionis [15], we have provided an approximate solution of Eq. (3.1), using  $f(z) \sim 1$  (which is valid at relatively large redshifts), but only in the framework of general relativity, i.e.,  $Y(t) = 1$ , which contains a quintessence (or phantom) dark energy. Here our aim is to provide a full analytical solution for all possible dark energy cosmologies that have appeared in the literature, such as a cosmological constant  $\Lambda$  (vacuum), time-varying  $w(t)$  cosmologies, quintessence,  $k$ -essence, quartessence, vector fields, phantom, modifications of gravity, Chaplygin gas, and so on.

Inserting now  $y(a) = g(a)/D(a)$  into Eq. (3.4) and using simultaneously Eq. (2.2) and the second equality of Eq. (3.5), we obtain

$$\frac{d^2 g}{da^2} + A(a) \frac{dg}{da} = 0. \quad (3.7)$$

That is, the general solution of the latter equation is

$$g(a) = C_1 + C_2 \int \frac{da}{a^3 E(a)}, \quad (3.8)$$

where  $C_1$  and  $C_2$  are the integration constants. Utilizing now  $a = (1+z)^{-1}$ ,  $b = y + 1 = (g/D) + 1$ ,  $b_0 = b(0)$ , and Eq. (3.8), we finally obtain the functional form that provides the evolution of linear bias for all possible types of DE models, including those of modified gravity, as

$$b(z) = 1 + \frac{b_0 - 1}{D(z)} + C_2 \frac{J(z)}{D(z)}, \quad (3.9)$$

where

$$J(z) = \int_0^z \frac{(1+x)dx}{E(x)}. \quad (3.10)$$

Because different halo masses result in different values of  $b_0$ , one should expect that the constants of integration  $C_1 = b_0 - 1$  and  $C_2$  should be functions of the mass of dark matter halos (see [16]), assuming that the extragalactic mass tracers are hosted by a dark matter halo of a given mass. Note that an extension of our model for the case of an evolving mass-tracer population (i.e., including the effects of halo merging) is provided in the Appendix.

Finally, comparing our solution of Eq. (3.9) with that of the usual galaxy-conserving bias evolution model,  $b(z) = 1 + (b_0 - 1)/D(z)$ , it becomes evident that the latter misses one of the two components of the full solution. Furthermore, our full solution does not suffer from the *unbiased* and the *low redshift* problems, but more importantly, the dependence of our bias evolution model on the

different cosmologies enters through the different behavior of  $D(a)$ , which is affected by  $\gamma$  [see Eq. (3.6)], and of  $E(a) = H(a)/H_0$ .

It is interesting to mention that measuring the growth index could provide an efficient way to discriminate between modified gravity models and DE models which adhere to general relativity. Indeed it was theoretically shown that for DE models inside general relativity the growth index  $\gamma$  is well fitted by  $\gamma_{\text{GR}} \approx 6/11$  (see [21,29]). Notice, that in the case of the braneworld model of Dvali, Gabadadze, and Porrati [30] (hereafter DGP) we have  $\gamma \approx 11/16$  (see also [21]). Indeed, it has been proposed (see [31]) that an efficient avenue to constrain the  $\gamma$  parameter is by determining observationally the redshift-dependent linear growth of perturbations. Alternatively other methods have been proposed in the literature, such as redshift space distortions in the galaxy power spectrum and the growth rate of massive galaxy clusters (see for example [32] and references therein). It is interesting to mention here that the above methods also assume a linear and scale-independent bias.

An alternative approach is to use the current generalized bias evolution, cosmology and  $\gamma$  dependent, relation and high-quality observational bias data to test gravity. Of course, the observational bias data are derived for a particular cosmological model, but it is an easy task to scale them to each tested model in a consistent manner. Note, that such data are already available in the literature for the case of optical quasistellar objects[33]. If the derived value of  $\gamma$  shows scale or time dependence or it is inconsistent with  $\gamma_{\text{GR}} \approx 6/11$ , then this will be a hint that the nature of

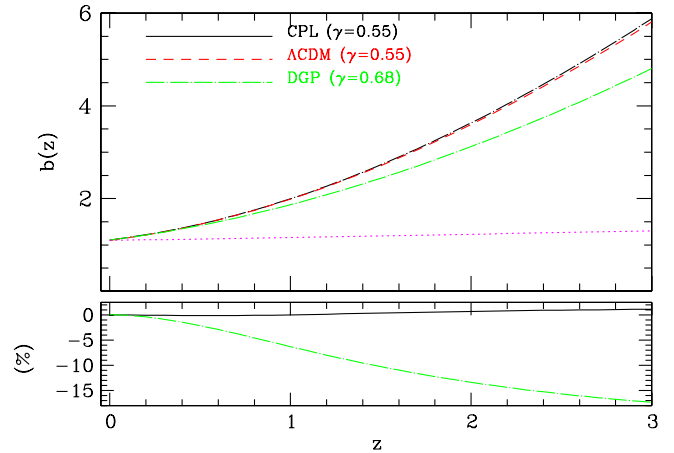


FIG. 1 (color online). The bias  $z$  evolution for different flat cosmological models (top) and their fractional difference with respect to the  $\Lambda$ CDM model (bottom). The models shown are CPL (solid line) with  $w(a) = w_0 + w_1(1 - a)$  and  $\gamma = 0.55$ , concordance  $\Lambda$ CDM (dashed line), and DGP (dotted-dashed line) with  $w(a) = -[1 + \Omega_m(a)]^{-1}$  and  $\gamma = 0.68$ . Note that we use  $\Omega_m = 0.27$ ,  $(w_0, w_1) = (-0.93, -0.38)$  [35],  $b_0 = 1.1$  and  $C_2 = 0.45$ . Finally, we also plot (dotted line) the bias evolution for  $C_2 = 0$ , which corresponds to that of [11].

dark energy reflects in the physics of gravity. Such an analysis is in progress and will be published elsewhere.

In order to visualize the redshift and  $\gamma$  dependence of our bias model, we compare in Fig. 1 a few flat cosmological models in which we impose  $\Omega_m = 0.27$ ,  $b_0 = 1.1$ , and  $C_2 = 0.45$ . In particular we consider the following cases:

- (a) the CPL parametrization [34] with  $\gamma = 0.55$  (solid line),
- (b) the concordance  $\Lambda$ CDM ( $\gamma = 0.55$ , dashed line), and
- (c) the DGP with  $\gamma = 0.68$  (dotted-dashed line).

The dotted line shows the bias evolution of the Tegmark and Peebles [11] model, which is also described by our bias model in the limit of  $C_2 = 0$ . In the lower panel of Fig. 1 we show the fractional difference of the model bias with respect to that of the  $\Lambda$ CDM.

#### IV. CONCLUSIONS

In this work we provide a general bias evolution model, based on linear perturbation theory, which is valid for all possible noninteracting dark energy models, including those of modified gravity. Thus the current generalization of the bias evolution model can be viewed as a necessary step and an ideal tool to test the validity of general relativity on cosmological scales.

It is, however, important to spell out clearly which are the basic assumptions of our model, which are common also to many bias models in the literature: (a) the mass tracers and the underlying mass share the same velocity/gravity field, (b) the biasing is linear on the scales of interest (which does not preclude being scale dependent on small nonlinear scales), and (c) that each dark matter halo is populated by one extragalactic mass tracer, which is an assumption that enters, at the present development of our model, only in the comparison of our model with observational bias data and not in the derivation of its functional form.

#### ACKNOWLEDGMENTS

We thank Joe Silk for useful comments. Manolis Plionis acknowledges funding by Mexican CONACyT Grant No. 2005-49878. S.B. wishes to thank the Department ECM of the University of Barcelona for its hospitality, and the financial support from the Spanish Ministry of Education, within the program of Estancias de Profesores e Investigadores Extranjeros en Centros Espanoles (SAB2010-0118).

#### APPENDIX A: BIAS EVOLUTION FOR AN EVOLVING MASS-TRACER POPULATION

Here we obtain the general linear bias evolution model assuming that the mass-tracer population evolves with time according to a  $(1+z)^\nu$  law. We now drop the assumption

used in Sec. III, that the mass-tracer number density is conserved in time, by allowing a contribution from the corresponding interactions among the mass tracers. We obtain again the corresponding Eq. (3.1), starting from the continuity equation and introducing an additional time-dependent term,  $\Psi(t)$ , which we associate with the effects of interactions and merging of the mass tracers. We also make the same assumption, as in our original formulation, that the tracers and the underlying mass distribution share the same velocity field (or gravity field). Then,

$$\dot{\delta}_m + \nabla u \simeq 0 \quad \text{and} \quad \dot{\delta}_{\text{tr}} + \nabla u + \Psi(t) \simeq 0, \quad (\text{A1})$$

from which we obtain

$$\dot{\delta}_m - \dot{\delta}_{\text{tr}} = \Psi. \quad (\text{A2})$$

Although we do not have a fundamental theory to model the time-dependent  $\Psi(t)$  function, it appears to depend on the tracer number density and its logarithmic derivative as well as on the tracer overdensity:  $\Psi(t) \propto \Psi(\bar{n}, (1 + \delta_{\text{tr}})d \ln \bar{n}/dt)$  (see eq. 10 of [14] and appendix of Basilakos *et al.* [16]).

Now, in the context of linear biasing, we have  $\delta_{\text{tr}} = b \delta_m$  and utilizing  $b = y + 1$ , we find that  $d(y\delta_m)/dt = -\Psi$ . Differentiating twice the latter we then get  $\ddot{y}\delta_m + 2\dot{y}\dot{\delta}_m + y\ddot{\delta}_m = -\dot{\Psi}$ . Solving for  $y\ddot{\delta}_m$ , using the fact that  $y\dot{\delta}_m = -\dot{y}\delta_m - \Psi$  and Eq. (2.1) we arrive at the following expression:

$$\ddot{y}\delta_m + 2(\dot{\delta}_m + H\delta_m)\dot{y} + 4\pi G_{\text{eff}}\rho_m\delta_{m,y} = -2H\Psi - \dot{\Psi}, \quad (\text{A3})$$

which is the corresponding Eq. (3.1) for the case of interactions among the tracers.

Transforming again Eq. (2.1) from  $t$  to  $a$ , we get

$$\frac{d^2 y}{da^2} + \left[ A(a) + \frac{2f(a)}{a} \right] \frac{dy}{da} + B(a)y = F(a), \quad (\text{A4})$$

where

$$F(a) = -\frac{2\Psi(a) + a(d\Psi/da)}{a^2 D(a)H(a)}. \quad (\text{A5})$$

Now, following the same notations ( $y = g/D$ ) as in Sec. III the above differential equation becomes

$$\frac{d^2 g}{da^2} + A(a)\frac{dg}{da} = F(a). \quad (\text{A6})$$

Integrating Eq. (A6) it is straightforward to estimate the general solution of the bias factor. This is

$$g(a) = C_1 + C_2 \int \frac{da}{a^3 E(a)} + \int \frac{da}{a^3 E(a)} \int^\alpha F(\tilde{a}) \tilde{a}^3 E(\tilde{a}) d\tilde{a}. \quad (\text{A7})$$

Using the same conditions with those provided in Sec. III, the bias evolution in the redshift space takes the form

$$b(z) = 1 + \frac{b_0 - 1}{D(z)} + C_2 \frac{J(z)}{D(z)} + \frac{y_p(z)}{D(z)}, \quad (\text{A8})$$

where

$$y_p(z) = \int_0^z \frac{(1+x)}{E(x)} dx \int_0^x \frac{F(u)E(u)}{(1+u)^5} du. \quad (\text{A9})$$

Obviously, if the interaction among the tracers is negligible ( $\Psi \simeq 0$ ) then Eq. (A8) boils down to Eq. (3.10) as it should.

Now, knowledge of the exact functional form of the interaction term  $\Psi(z)$  would provide the precise redshift evolution of the bias. As we have analytically proved in the appendix of Basilakos *et al.* [16], a reasonable approach regarding the evolutionary  $\Psi(z)$  term is that  $\Psi(z) = AH_0(1+z)^\nu$ , where  $\nu \sim 3$ . Note that the Hubble constant has been maintained for mathematical convenience. Inserting the latter equation,  $a = (1+z)^{-1}$  and

$d\Psi/da = -(1+z)^2 d\Psi/dz$  into the second term of Eq. (A5), we derive that

$$F(z) = A(\nu - 2) \frac{(1+z)^{\nu+2}}{D(z)E(z)}, \quad (\text{A10})$$

where  $A$  is a positive parameter (to be determined from observational data in a forthcoming paper). Obviously, for  $\nu > 2$  the derived bias evolution becomes stronger than in the case of no interactions, especially at high redshifts, which means that due to the merging processes the halos (of some particular mass) correspond to higher peaks of the underlying density field with respect to equal mass halos in the noninteracting case. On the other hand, the  $\nu < 2$  case corresponds to the destruction of halos of a particular mass, which results into a lower rate of bias evolution with respect to the noninteracting case. Now, for the limiting case with  $\nu = 2$  we obtain  $y_p = 0$ , implying no contribution of the interacting term to the bias evolution solution, as in the case with  $\Psi = 0$ , which can be interpreted as the case where the destruction and creation processes are counterbalanced.

- 
- [1] N. Kaiser, *Astrophys. J.* **284**, L9 (1984).  
[2] J.M. Bardeen, J.R. Bond, N. Kaiser, and A.S. Szalay, *Astrophys. J.* **304**, 15 (1986).  
[3] W.H. Press and P. Schechter, *Astrophys. J.* **187**, 425 (1974).  
[4] S. Cole and N. Kaiser, *Mon. Not. R. Astron. Soc.* **237**, 1127 (1989); H.J. Mo and S.D.M. White, *Mon. Not. R. Astron. Soc.* **282**, 347 (1996); S. Matarrese, P. Coles, F. Lucchin, and L. Moscardini, *Mon. Not. R. Astron. Soc.* **286**, 115 (1997); L. Moscardini, P. Coles, F. Lucchin, and S. Matarrese, *Mon. Not. R. Astron. Soc.* **299**, 95 (1998); R.K. Sheth and G. Tormen, *Mon. Not. R. Astron. Soc.* **308**, 119 (1999); P. Valageas, *Astron. Astrophys.* **508**, 93 (2009); **525**, A98 (2011).  
[5] R.K. Sheth and G. Tormen, *Mon. Not. R. Astron. Soc.* **323**, 1 (2001).  
[6] Y.P. Jing, *Astrophys. J.* **503**, L9 (1998); J. Tinker, D.H. Weinberg, Z. Zheng, and I. Zehavi, *Astrophys. J.* **631**, 41 (2005).  
[7] U. Seljak and M. Warren, *Mon. Not. R. Astron. Soc.* **355**, 129 (2004); J. Tinker *et al.*, *Astrophys. J.* **724**, 878 (2010); A. Pillepich, C. Porciani, and O. Han, *Mon. Not. R. Astron. Soc.* **402**, 191 (2010).  
[8] Chung-Pei Ma, M. Maggiore, A. Riotto, and J. Zhang, *Mon. Not. R. Astron. Soc.* **411**, 2644 (2011).  
[9] M. Nusser and M. Davis, *Astrophys. J.* **421**, L1 (1994).  
[10] J.N. Fry, *Astrophys. J.* **461**, 65 (1996); P. Catelan, F. Lucchin, S. Matarrese, and C. Porciani, *Mon. Not. R. Astron. Soc.* **297**, 692 (1998).  
[11] M. Tegmark and P.J.E. Peebles, *Astrophys. J.* **500**, L79 (1998).  
[12] L. Hui and K.P. Parfrey, *Phys. Rev. D* **77**, 043527 (2008); B.M. Schaefer, M. Douspis, and N. Aghanim, *Mon. Not. R. Astron. Soc.* **397**, 925 (2009).  
[13] J.S. Bagla, *Mon. Not. R. Astron. Soc.* **299**, 417 (1998).  
[14] P. Simon, *Astron. Astrophys.* **430**, 827 (2005).  
[15] S. Basilakos and M. Plionis, *Astrophys. J.* **550**, 522 (2001); **593**, L61 (2003).  
[16] S. Basilakos, M. Plionis, and C. Ragone-Figueroa, *Astrophys. J.* **678**, 627 (2008).  
[17] A. Lue, R. Scossimarro, and G.D. Starkman, *Phys. Rev. D* **69**, 124015 (2004).  
[18] E.V. Linder, *Phys. Rev. D* **72**, 043529 (2005).  
[19] F.H. Stabenau and B. Jain, *Phys. Rev. D* **74**, 084007 (2006).  
[20] P.J. Uzan, *Gen. Relativ. Gravit.* **39**, 307 (2007).  
[21] E.V. Linder, *Phys. Rev. D* **70**, 023511 (2004); E.V. Linder and R.N. Cahn, *Astropart. Phys.* **28**, 481 (2007).  
[22] S. Tsujikawa, K. Uddin, and R. Tavakol, *Phys. Rev. D* **77**, 043007 (2008).  
[23] R. Gannouji, B. Moraes, and D. Polarski, *J. Cosmol. Astropart. Phys.* **02** (2009) 034.  
[24] P.J.E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, Princeton, New Jersey, 1993).  
[25] V. Silveira and I. Waga, *Phys. Rev. D* **50**, 4890 (1994).  
[26] L. Wang and J.P. Steinhardt, *Astrophys. J.* **508**, 483 (1998).  
[27] S. Basilakos, *Astrophys. J.* **590**, 636 (2003).  
[28] P. McDonald and A. Roy, *J. Cosmol. Astropart. Phys.* **08** (2009) 20; D.S. Reed *et al.*, *Mon. Not. R. Astron. Soc.* **394**, 624 (2009).

- [29] S. Nesseris and L. Perivolaropoulos, *Phys. Rev. D* **77**, 023504 (2008).
- [30] G. Dvali, G. Gabadadze, and M. Porrati, *Phys. Lett. B* **485**, 208 (2000).
- [31] A. Vikhlinin *et al.*, arXiv:0903.5320.
- [32] L. Guzzo *et al.*, *Nature (London)* **451**, 541 (2008); E. Linder, *Astropart. Phys.* **29**, 336 (2008); D. Rapetti, S. W. Allen, A. Mantz, and H. Ebeling, *Mon. Not. R. Astron. Soc.* **406**, 1796 (2010).
- [33] S.M. Croom *et al.*, *Mon. Not. R. Astron. Soc.* **356**, 415 (2005); Y. Shen *et al.*, *Astrophys. J.* **697**, 1656 (2009); N.P. Ross *et al.*, *Astrophys. J.* **697**, 1634 (2009).
- [34] M. Chevallier and D. Polarski, *Int. J. Mod. Phys. D* **10**, 213 (2001); E. V. Linder, *Phys. Rev. Lett.* **90**, 091301 (2003).
- [35] E. Komatsu *et al.*, *Astrophys. J.* **192**, 18 (2011).