

Dynamics and constraints of the massive graviton dark matter flat cosmologies

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We discuss the dynamics of the Universe within the framework of the massive graviton cold dark matter scenario (MGCDM) in which gravitons are geometrically treated as massive particles. In this modified gravity theory, the main effect of the gravitons is to alter the density evolution of the cold dark matter component in such a way that the Universe evolves to an accelerating expanding regime, as presently observed. Tight constraints on the main cosmological parameters of the MGCDM model are derived by performing a joint likelihood analysis involving the recent supernovae type Ia data, the cosmic microwave background shift parameter, and the baryonic acoustic oscillations as traced by the Sloan Digital Sky Survey red luminous galaxies. The linear evolution of small density fluctuations is also analyzed in detail. It is found that the growth factor of the MGCDM model is slightly different ($\sim 1\text{--}4\%$) from the one provided by the conventional flat Λ CDM cosmology. The growth rate of clustering predicted by MGCDM and Λ CDM models are confronted to the observations and the corresponding best fit values of the growth index (γ) are also determined. By using the expectations of realistic future x-ray and Sunyaev-Zeldovich cluster surveys we derive the dark matter halo mass function and the corresponding redshift distribution of cluster-size halos for the MGCDM model. Finally, we also show that the Hubble flow differences between the MGCDM and the Λ CDM models provide a halo redshift distribution departing significantly from the those predicted by other dark energy models. These results suggest that the MGCDM model can observationally be distinguished from Λ CDM and also from a large number of dark energy models recently proposed in the literature.

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I. INTRODUCTION

The high-quality cosmological observational data (e.g. supernovae type Ia, cosmic microwave background (CMB), galaxy clustering, etc.), accumulated during the last two decades, have enabled cosmologists to gain substantial confidence that modern cosmology is capable of quantitatively reproducing the details of many observed cosmic phenomena, including the late-time accelerating stage of the Universe. Studies by many authors have converged to a cosmic expansion history involving a spatially flat geometry and a cosmic dark sector formed by cold dark matter and some sort of dark energy, endowed with large negative pressure, in order to explain the observed accelerating expansion of the Universe [1–8].

In spite of that, the absence of a fundamental physical theory, regarding the mechanism inducing the cosmic acceleration, has given rise to a plethora of alternative cosmological scenarios. Most are based either on the existence of new fields in nature (dark energy) or in some modification of Einstein's general relativity, with the present accelerating stage appearing as a sort of geometric effect.

The simplest dark energy candidate corresponds to a cosmological constant, Λ (see [9] for reviews). In the standard concordance cosmological (Λ CDM) model, the overall cosmic fluid contains baryons, cold dark matter plus a vacuum energy. This model fits accurately the current observational data and it therefore provides an excellent scenario to describe the observed Universe. However, it is well known that the concordance model suffers from, among others [10], two fundamental problems:

- (i) *Fine-tuning problem*—the fact that the observed value of the vacuum energy density ($\rho_\Lambda = \Lambda c^2/8\pi G \simeq 10^{-47} \text{ GeV}^4$) is more than 120 orders

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of magnitude below the natural value estimated using quantum field theory [11].

- (ii) *Coincidence problem*—the fact that the matter and the vacuum energy densities are of the same order just prior to the present epoch [12].

Such problems have inspired many authors to propose alternative dark energy candidates such as $\Lambda(t)$ cosmologies, quintessence, k -essence, vector fields, phantom dark energy, tachyons, Chaplygin gas, and the list goes on (see [13–28] and references therein). Naturally, in order to establish the evolution of the dark energy equation of state (EoS), a realistic form of $H(a)$ is required which should be constrained through a combination of independent dark energy probes.

Nevertheless, there are other possibilities to explain the present accelerating stage. For instance, one may consider that the dynamical effects attributed to dark energy can be mimicked by a nonstandard gravity theory. In other words, the present accelerating stage of the Universe can also be driven only by cold dark matter under a modification of the nature of gravity. Such a reduction of the so-called dark sector is naturally obtained in the so-called $f(R)$ gravity theories [29] (see, however, [30]).

On the other hand, general relativity predicts that gravitational waves are nondispersive and propagate with the same vacuum light speed. These results lead to the common belief that the graviton (the “boson” for general relativity), must be a massless particle. However, massive gravitons are features of some alternatives to general relativity such as the one proposed by Visser [31]. Such theories have motivated many experiments and observations in order to detect a possible dispersive behavior due to a nonzero graviton mass (see [32] and references therein).

More recently, it was shown that the massive graviton approach proposed by Visser can be used to build realistic cosmological models that can then be tested against the available cosmological data [33]. One of the main advantages of such massive graviton cosmology is the fact that it contains the same number of free parameters as the concordance Λ CDM model, and, therefore, it does not require the introduction of any extra fields in its dynamics. In this way, since the astronomical community is planning a variety of large observational projects intended to test and constrain the standard Λ CDM concordance model, as well as many of the proposed alternative models, it is timely and important to identify and explore a variety of physical mechanisms (or substances) which could also be responsible for the late-time acceleration of the Universe.

In what follows we focus our attention to a cosmological model within Visser’s massive graviton theory. In particular we discuss how to differentiate the massive graviton model from the concordance Λ CDM model. Initially, a joint statistical analysis, involving the latest observational data, SNIa, CMB shift parameter, and baryonic acoustic

oscillations (BAO), are implemented. Second, we attempt to discriminate the MGCDM and Λ CDM models by computing the halo mass function and the corresponding redshift distribution of the cluster-size halos. Finally, by using future x-ray and Sunayev-Zeldovich (SZ) surveys we show that the evolution of the cluster abundances is a potential discriminator between the MGCDM and Λ CDM models. We would like to stress here that the abundance of collapsed structures, as a function of mass and redshift, is a key statistical test for studies of the matter distribution in the Universe, and, more importantly, it can be accessed through observations [34]. Indeed, the mass function of galaxy clusters has been measured based on x-ray surveys [35–37], via weak and strong lensing studies [38–40], using optical surveys, like the Sloan Digital Sky Survey (SDSS) [41,42], as well as through the SZ effect [43]. In the last decade many authors have been involved in these kinds of studies and have found that the abundance of the collapsed structures is affected by the presence of a dark energy component [44–56].

The paper is planned as follows. The basic elements of Visser’s theory are presented in Sec. II, where we also introduce the cosmological equations for a flat Friedmann-Lemaître-Robertson-Walker (FLRW) geometry with massive gravitons. In Sec. III, a joint statistical analysis based on SNe Ia, CMB, and BAO is used to constraint the massive graviton cosmological model free parameter. The linear growth factor of matter perturbations is discussed in Sec. IV, while in Sec. V, we discuss and compare the corresponding theoretical predictions regarding the evolution of the cluster abundances. Finally, the main conclusions are summarized in Sec. VI.

II. MASSIVE GRAVITON COLD DARK MATTER COSMOLOGY: BASIC EQUATIONS

In this section we briefly present the main points of Visser’s massive gravity approach [31]. The full action is given by (in what follows $\hbar = c = 1$)

$$S = \int d^4x \left[\sqrt{-g} \frac{R(g)}{16\pi G} + \mathcal{L}_{\text{mass}_g}(g, g_0) + \mathcal{L}_{\text{matter}}(g) \right], \quad (2.1)$$

where besides the Einstein-Hilbert Lagrangian and the Lagrangian of the matter fields, we have the bimetric Lagrangian:

$$\begin{aligned} \mathcal{L}_{\text{mass}}(g, g_0) = & \frac{1}{2} m_g^2 \sqrt{-g_0} \left\{ (g_0^{-1})^{\mu\nu} (g - g_0)_{\mu\sigma} (g_0^{-1})^{\sigma\rho} \right. \\ & \left. \times (g - g_0)_{\rho\nu} - \frac{1}{2} [(g_0^{-1})^{\mu\nu} (g - g_0)_{\mu\nu}]^2 \right\}, \end{aligned} \quad (2.2)$$

where m_g is the graviton mass and $(g_0)_{\mu\nu}$ is a general flat metric.

The field equations, which are obtained by variation of (2.1), can be written as

$$G^{\mu\nu} - \frac{1}{2}m_g^2 M^{\mu\nu} = -8\pi G T^{\mu\nu}, \quad (2.3)$$

where $G^{\mu\nu}$ is the Einstein tensor, $T^{\mu\nu}$ is the energy-momentum tensor for perfect fluid, and the contribution of the massive tensor to the field equations reads

$$M^{\mu\nu} = (g_0^{-1})^{\mu\sigma}[(g - g_0)_{\sigma\rho} - \frac{1}{2}(g_0)_{\sigma\rho}(g_0^{-1})^{\alpha\beta} \times (g - g_0)_{\alpha\beta}](g_0^{-1})^{\rho\nu}. \quad (2.4)$$

Note that if one takes the limit $m_g \rightarrow 0$, the standard Einstein field equations are recovered.

Thus, from the construction of Visser's theory, it can be classified as a bimetric theory of gravitation. This kind of theory was first studied by Rosen [57]. In Rosen's concept the metric $g_{\mu\nu}$ describes the geometry of spacetime in the same way as in the context of the general relativity theory, and the second metric $(g_0)_{\mu\nu}$ (that Rosen denoted by $\gamma_{\mu\nu}$) refers to the flat spacetime and describes the inertial forces. It is worth mentioning that Rosen has shown that a bimetric theory satisfies the covariance and the equivalence principles, a fact that was also pointed out by Visser. In this way, in order to follow Rosen's approach, we have constrained the background metric to respect the Riemann-flat condition, that is, $R_{\mu\nu\kappa}^\lambda(g_0) \equiv 0$ in such a way that we have no ambiguity on the choice of $(g_0)_{\mu\nu}$, it will always be chosen to be a flat metric, depending only on the particular coordinates we are dealing with, of course.

Regarding the energy-momentum conservation we will follow the same approach as that of Refs. [58,59]. Since the Einstein tensor satisfies the Bianchi identities $\nabla_\nu G^{\mu\nu} = 0$, the energy conservation law is expressed as

$$\nabla_\nu T^{\mu\nu} = \frac{m_g^2}{16\pi G} \nabla_\nu M^{\mu\nu}. \quad (2.5)$$

In the above framework, the global dynamics of a flat Λ CDM cosmology is driven by the following equations:¹

$$8\pi G\rho = 3\left(\frac{\dot{a}}{a}\right)^2 + \frac{3}{4}m_g^2(a^2 - 1), \quad (2.6)$$

$$8\pi Gp = -2\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{4}m_g^2 a^2(a^2 - 1), \quad (2.7)$$

where ρ is the energy density, p is the pressure, and $a(t)$ is the scale factor.

¹In the present article we restrict our analysis to the flat cosmologies in order to compare our results with those of the flat Λ CDM model, which is the most accepted cosmological model as shown, e.g., by the WMAP7 data [5]. A generalization of the model for a non-spatially flat cosmology will appear in a forthcoming article.

From Eq. (2.5) we get the evolution equation for the energy density, namely,

$$\dot{\rho} + 3H\left[(\rho + p) + \frac{m_g^2}{32\pi G}(a^4 - 6a^2 + 3)\right] = 0, \quad (2.8)$$

where $H = \dot{a}/a$. By integrating the above equation for a matter dominated universe ($p = 0$), one obtains:

$$\rho(a) = \frac{\rho_0}{a^3} - \frac{3m_g^2}{32\pi G}\left(\frac{a^4}{7} - \frac{6a^2}{5} + 1\right), \quad (2.9)$$

where ρ_0 is the present value of the energy density. As expected, in the limiting case $m_g \rightarrow 0$ all the standard FLRW expressions are recovered.

Now, inserting (2.9) in the modified Friedmann equation (2.6) we obtain the normalized Hubble parameter

$$E^2(a) = \frac{H^2(a)}{H_0^2} = \Omega_m a^{-3} + \delta H^2, \quad (2.10)$$

with

$$\delta H^2 = \frac{1}{2}\Omega_g(7a^2 - 5a^4), \quad (2.11)$$

where H_0 is the Hubble constant, Ω_m is the matter density parameter (for baryons and dark matter $\Omega_i = \rho_{i0}/\rho_{c0}$, where $\rho_{c0} = 3H_0^2/8\pi G$ is the critical density parameter), and $\Omega_g = \frac{1}{70}\left(\frac{m_g}{H_0}\right)^2$ is the present contribution of the massive gravitons. It should be stressed that the last term of the above normalized Hubble function (2.10) encodes the correction to the standard FLRW expression.

In general, using the FLRW equations, one can express the effective dark energy EoS parameter in terms of the normalized Hubble parameter [60]

$$w_{\text{DE}}(a) = \frac{-1 - \frac{2}{3}a\frac{d\ln E}{da}}{1 - \Omega_m a^{-3} E^{-2}(a)}. \quad (2.12)$$

After some simple algebra, it is also readily seen that the effective ("geometrical" in our case) dark energy EoS parameter is given by (see [25,61])

$$w_{\text{DE}}(a) = -1 - \frac{1}{3}\frac{d\ln\delta H^2}{d\ln a}. \quad (2.13)$$

In our case, inserting Eq. (2.11) into Eq. (2.13) it is straightforward to obtain a simple analytical expression for the geometrical dark energy EoS parameter

$$w_{\text{DE}}(a) = -1 - \frac{2}{3}\left(\frac{7 - 10a^2}{7 - 5a^2}\right). \quad (2.14)$$

It thus follows that in the cosmological context, the modified gravity theory as proposed by Visser can be treated as an additional effective fluid with the EoS parameter defined by (2.14). Note also that the current Hubble function has only two free parameters (H_0 and Ω_m), exactly the same number of free parameters as the conventional flat Λ CDM model. Naturally, the value of H_0

is not predicted by any of the models, and it is set to its observational value of $H_0 = 70.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [5,62].

III. LIKELIHOOD ANALYSIS

Let us now discuss the statistical treatment of the observational data used to constrain the MGCDM model presented in the previous section.

To begin with, we consider the *Constitution* supernovae Ia set of Hicken *et al.* [6], but in order to avoid possible problems related to the local bulk flow, we use a subset of this sample containing 366 SNe Ia all with redshifts $z > 0.02$. The likelihood estimator is determined by a χ^2_{SNIa} statistics:

$$\chi^2_{\text{SNIa}}(\Omega_m) = \sum_{i=1}^{366} \left[\frac{\mu^{\text{th}}(a_i, \Omega_m) - \mu^{\text{obs}}(a_i)}{\sigma_i} \right]^2, \quad (3.1)$$

where $a_i = (1 + z_i)^{-1}$ is the scale factor of the Universe at the observed redshift z_i , μ is the distance modulus $\mu = m - M = 5 \log d_L + 25$ and d_L is the luminosity distance,² $d_L(a, \Omega_m) = ca^{-1} \int_a^1 \frac{dv}{y^2 H(v)}$. Now, from the likelihood analysis we find that $\Omega_m = 0.266 \pm 0.016$ with $\chi^2_{\text{tot}}(\Omega_m)/\text{dof} \approx 446.5/365$.

In addition to the SNe Ia data, we also consider the BAO scale produced in the last scattering surface by the competition between the pressure of the coupled baryon-photon fluid and gravity. The resulting acoustic waves leave (in the course of the evolution) an overdensity signature at certain length scales of the matter distribution. Evidence of this excess was recently found in the clustering properties of SDSS galaxies (see [63–65]) and it provides a suitable “standard ruler” for constraining dark energy models. In this work we use the measurement derived by Einstein *et al.* [63]. In particular, we utilize the following estimator $A(\Omega_m) = \frac{\sqrt{\Omega_m}}{[z_s^2 E(a_s)]^{1/3}} \left[\int_{a_s}^1 \frac{da}{a^2 E(a)} \right]^{2/3}$, measured from the SDSS data to be $A = 0.469 \pm 0.017$, where $z_s = 0.35$ [or $a_s = (1 + z_s)^{-1} \approx 0.75$]. Therefore, the corresponding χ^2_{BAO} function can be written as:

$$\chi^2_{\text{BAO}}(\Omega_m) = \frac{[A(\Omega_m) - 0.469]^2}{0.017^2}. \quad (3.2)$$

The likelihood function peaks at $\Omega_m = 0.306^{+0.026}_{-0.025}$.

Finally, a very interesting geometrical probe of dark energy is provided by the angular scale of the sound horizon at the last scattering surface. It is encoded in the location of the first peak of the angular (CMB) power spectrum [66,67], and may be defined by the quantity $\mathcal{R} = \sqrt{\Omega_m} \int_{a_{ls}}^1 \frac{da}{a^2 E(a)}$. The shift parameter measured from the WMAP 7-year data [5] is $\mathcal{R} = 1.726 \pm 0.019$ at

$z_{ls} = 1091.36$ [or $a_{ls} = (1 + z_{ls})^{-1} \approx 9.154 \times 10^{-4}$]. In this case, the χ^2_{cmb} function reads

$$\chi^2_{\text{cmb}}(\Omega_m) = \frac{[\mathcal{R}(\Omega_m) - 1.726]^2}{0.018^2}. \quad (3.3)$$

It should be stressed that for CMB shift parameter, the contribution of the radiative component, ($\Omega_R a^{-4}$, where $\Omega_R \approx 4.174 \times 10^{-5} h^{-2}$) needs also to be considered [5]. Note also that the measured CMB shift parameter is somewhat model dependent but such details of the models were not included in our analysis. For example, such is the case when massive neutrinos are included. The robustness of the shift parameter has been tested and discussed in [68]. In this case the best fit value is $\Omega_m = 0.263 \pm 0.03$.

The derived Ω_m values from each individual probe appear to be quite different, although within their mutual 2σ uncertainty range. Therefore, in order to put tighter constraints on the corresponding parameter space of any cosmological model, the above probes are combined through a joint likelihood analysis,³ given by the product of the individual likelihoods according to: $\mathcal{L}_{\text{tot}}(\Omega_m) = \mathcal{L}_{\text{SNIa}} \times \mathcal{L}_{\text{BAO}} \times \mathcal{L}_{\text{cmb}}$, which translates in the joint χ^2 function in an addition: $\chi^2_{\text{tot}}(\Omega_m) = \chi^2_{\text{SNIa}} + \chi^2_{\text{BAO}} + \chi^2_{\text{cmb}}$.

Now, by applying our joint statistical procedure for both cosmologies, we obtain the following best fit parameters:

- (i) MGCDM model: $\Omega_m = 0.276 \pm 0.012$ with $\chi^2_{\text{tot}}(\Omega_m)/\text{dof} \approx 448.5/367$. Such results should be compared to those found by Alves *et al.* [33], namely, $\Omega_m = 0.273 \pm 0.015$ with a $\chi^2_{\text{tot}}(\Omega_m)/\text{dof} \approx 565.06/558$. This difference must be probably attributed to the use of the *Union2* supernovae sample [69] by the latter authors.
- (ii) Λ CDM model: $\Omega_m = 0.280 \pm 0.010$ with $\chi^2_{\text{tot}}(\Omega_m)/\text{dof} \approx 439.5/367$, which is in good agreement with recent studies [1–8].

It should be mentioned here that using the BAO results of Percival *et al.* [64], does not change the previously presented constraints.

IV. MGCDM VERSUS Λ CDM COSMOLOGY

A. The cosmic expansion history

In Fig. 1 we plot the normalized MGCDM Hubble function (solid line) as a function of redshift, which appears quite different both in amplitude and shape with respect to the corresponding Λ CDM model expectations (dashed line).

In Fig. 2 (upper panel), we present the evolution of the MGCDM effective dark energy EoS parameter. One can divide the evolution of the cosmic expansion history in

²Since only the relative distances of the SNIa are accurate and not their absolute local calibration, we always marginalize with respect to the internally derived Hubble constant.

³Likelihoods are normalized to their maximum values. In the present analysis we always report 1σ uncertainties on the fitted parameters. Note also that the total number of data points used here is $N_{\text{tot}} = 368$, while the associated degrees of freedom are $\text{dof} = 367$. Note that we sample $\Omega_m \in [0.1, 1]$ in steps of 0.001.

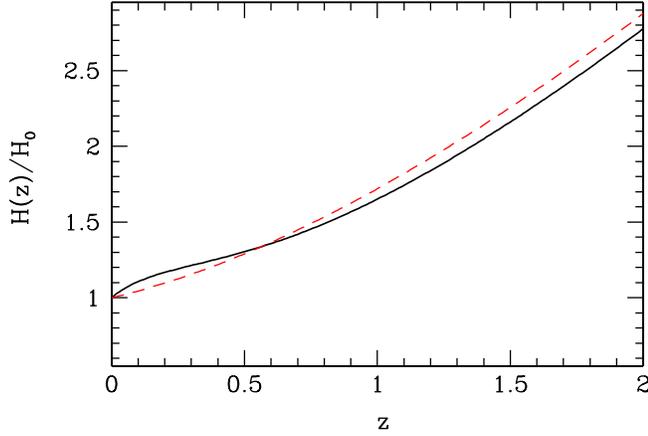


FIG. 1 (color online). Normalized Hubble parameter as a function of redshift. The solid line is the prediction of the MGCDM model. For comparison, the dashed line corresponds to the traditional Λ CDM model.

different phases on the basis of the varying behavior of the MGCDM and Λ CDM models. We will investigate such variations in terms of the deceleration parameter, $q(a) = -(1 + d \ln H / d \ln a)$, which is plotted in the lower panel of Fig. 2. In the inset plot we display the relative deviation of the deceleration parameter, $\Delta(q - q_\Lambda)$, between the two cosmological models. We can divide the cosmic expansion history in the following phases:

- (i) At early enough times $a \leq 0.1$ the deceleration parameters of both models are positive with $q \approx q_\Lambda$, which means that the two cosmological models provide a similar expansion rate of the Universe. Note that by taking the limit $\lim_{a \rightarrow 0} w_{\text{DE}}(a) = -5/3$ for the MGCDM model, while we always have $w_{\text{DE}} = -1$ for the Λ CDM model.
- (ii) For $0.1 \leq a \leq 0.44$ the deceleration parameters are both positive with $q > q_\Lambda$, which means that the cosmic expansion in the MGCDM model is more rapidly “decelerating” than in the Λ CDM case.
- (iii) Between $0.44 < a < 0.52$ the deceleration parameters remain positive but $q < q_\Lambda$.
- (iv) For $0.52 \leq a \leq 0.57$ the traditional Λ model remains in the decelerated regime ($q_\Lambda > 0$) but the MGCDM is starting to accelerate ($q < 0$).
- (v) For $0.57 < a \leq 0.94$ the deceleration parameters are both negative and since $q < q_\Lambda$, the MGCDM model provides a stronger acceleration than in the Λ CDM model (the opposite situation holds at $0.85 \leq a \leq 0.94$).

Interestingly, prior to the present epoch ($a > 0.94$) the deceleration parameter of the MGCDM model becomes positive and when $a = 1$, we have $w_{\text{DE}} = 0$, i.e. the Universe becomes again matter dominated, implying that the late-time acceleration of the Universe was a transient phase which has already finished.

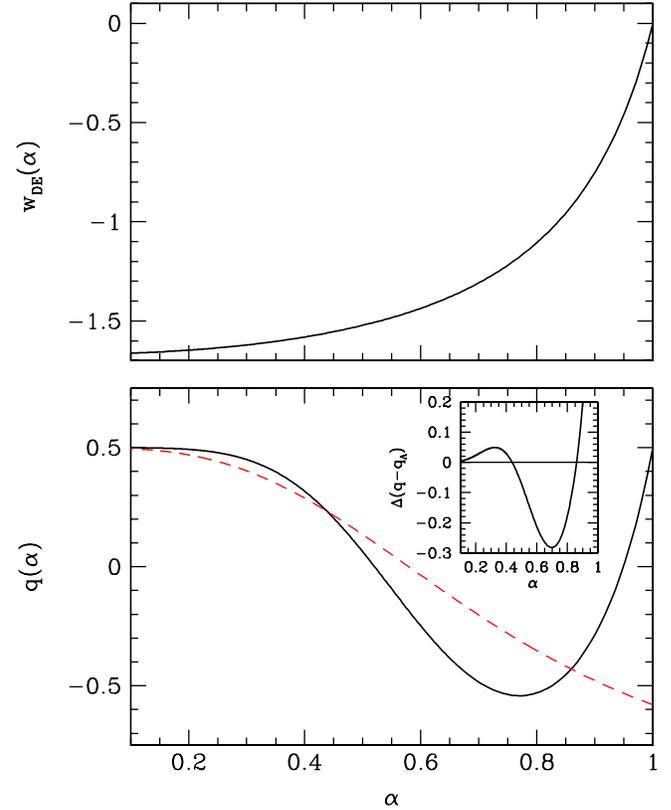


FIG. 2 (color online). Expansion history. In the upper panel we display the evolution of the dark energy effective EoS parameter. In the lower panel we compare the deceleration parameters of the MGCDM (solid line) and the concordance Λ CDM (dashed line) models. In the inset we show the relative deviation $\Delta(q - q_\Lambda)$ of the two deceleration parameters.

From the inset panel of Fig. 2 it becomes clear that the MGCDM model reaches a maximum deviation from the Λ CDM cosmology prior to $a \approx 0.75$ and again at $a \approx 1$. Finally, the deceleration parameters at the present time are $q_0 \approx 0.50$ and $q_{0\Lambda} \approx -0.58$. If we go further to the future we find from Eq. (2.14) that the state parameter as well as the deceleration parameter diverges for $a = \sqrt{7/5}$. This value sets the turning point after which the Universe begins to contract in the MGCDM model (for more details see [70].)

B. The growth factor and the rate of clustering

It is well known that for small scales (smaller than the horizon) the dark energy component (or “geometrical” dark energy) is expected to be smooth and thus it is fair to consider perturbations only on the matter component of the cosmic fluid [71]. This assumption leads to the usual equation for matter perturbations

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\text{eff}}\rho_m\delta_m = 0, \quad (4.1)$$

where the effect of "geometrical" dark energy is introduced via the expression of $G_{\text{eff}} = G_{\text{eff}}(t)$ (see [72,73]). In the context of general relativity G_{eff} coincides with the Newton's gravitational constant. Now, for any type of dark energy an efficient parametrization of the matter perturbations ($\delta_m \propto D$) is based on the growth rate $f(a) \equiv d \ln D / d \ln a$ [74], which has the following functional form:

$$f(a) = \frac{d \ln D}{d \ln a} = \Omega_m^\gamma(a), \quad (4.2)$$

where $D(a)$ is the linear growth factor, $\Omega_m(a) = \Omega_m a^{-3} / E^2(a)$ and γ is the so-called growth index (see Refs. [25,61,72,75,76]). Since the growth factor of a pure matter universe (Einstein de-Sitter) has the form $D_{\text{EdS}} = a$, one has to normalize the different cosmological models such that $D \simeq a$ at large redshifts due to the dominance of the nonrelativistic matter component. Using the latter condition we can easily integrate Eq. (4.2) to derive the growth factor [25]

$$D(a) = a e^{\int_0^a (dx/x) [\Omega_m^\gamma(x) - 1]}. \quad (4.3)$$

In the present case we are working with a modification of Einstein's gravity instead of an extra fluid, in such a way the usual Poisson equation for the gravitational potential is modified due the presence of the mass term. In the simple case of the nonrelativistic limit we have a Yukawa-like potential which accomplishes corrections to the Newtonian potential to scales of the order of the Compton wavelength of the graviton, $\lambda = m_g^{-1}$. Using this kind of potential, the classic limit for the graviton mass obtained from solar system dynamics observations is $m_g < 7.68 \times 10^{-55}$ g [77], but one of the most stringent constraints is obtained by requiring the derived dynamical properties of a galactic disk to be consistent with observations [59] thereby yielding $m_g < 10^{-59}$ g. Now, by considering the best fit value obtained here for Ω_g we have $m_g \sim 10^{-65}$ g, which is nearly 6 orders of magnitude below to the previous bound. This value gives a Compton wavelength of the order of the horizon. The Compton wavelength can be seen as the physical length of graviton's perturbations which implies that these perturbations play some role only close to the Hubble radius and thus they will be negligible at subhorizon scales. In other words, Eqs. (4.1) and (4.3) are both valid also in the MGCDM model.

Clearly in order to quantify the evolution of the growth factor we need to know the growth index. Since for the current graviton model there is yet no theoretically predicted value of growth index, we attempt to provide a relevant value by performing a standard χ^2 minimization procedure (described previously) between the observationally measured growth rate (based on the 2dF and SDSS galaxy catalogs; see Table I; [78]) and that expected in the MGCDM cosmological model, according to

TABLE I. Data of the growth rate of clustering [78]. The correspondence of the columns is as follows: redshift, observed growth rate, and references.

z	f_{obs}	Refs.
0.15	0.51 ± 0.11	[79,80]
0.35	0.70 ± 0.18	[81]
0.55	0.75 ± 0.18	[82]
1.40	0.90 ± 0.24	[83]
3.00	1.46 ± 0.29	[84]

$$\chi^2(\gamma) = \sum_{i=1}^5 \left[\frac{f_{\text{obs}}(z_i) - f_{\text{model}}(z_i, \gamma)}{\sigma_i} \right]^2, \quad (4.4)$$

where σ_i is the observed growth rate uncertainty. Note that for comparison we perform the same analysis also for the Λ CDM model.

In Fig. 3 (upper panel), we present the measured $f_{\text{obs}}(z)$ (filled symbols) with the estimated growth rate function, $f(z) = \Omega_m^\gamma(z)$, for the two considered cosmological models. Notice, that for the MGCDM cosmological model (solid line) we use $\Omega_m = 0.276$ and for the Λ case (dashed line) $\Omega_m = 0.280$, which are the values provided by our likelihood analysis of Sec. III. In the inset panel of Fig. 3 we plot the variation of $\Delta\chi^2 = \chi^2(\gamma) - \chi_{\text{min}}^2(\gamma)$ around the best γ fit value. For the MGCDM model we find $\gamma = 0.56_{-0.14}^{+0.15}$ ($\chi^2/\text{dof} \simeq 0.69$), while for the Λ CDM model we obtain $\gamma_\Lambda = 0.62_{-0.15}^{+0.18}$ ($\chi^2/\text{dof} \simeq 0.75$), which is somewhat greater, but within 1σ , of the theoretically predicted value of $\gamma_\Lambda \simeq 6/11$. Such a discrepancy between the theoretical Λ CDM and observationally fitted value of γ has also been found by other authors. For example, Di Porto and Amendola [85] obtained $\gamma = 0.60_{-0.30}^{+0.40}$, while Nesseris and Perivolaropoulos [78], based on mass fluctuations inferred from independent observations at different redshifts, found $\gamma = 0.67_{-0.17}^{+0.20}$. If such a systematic difference between the measured and the theoretical γ Λ CDM values is due to observational uncertainties or the method used to estimate the observed γ , then one may expect a similar systematic difference to affect the measured γ value for the MGCDM model, pointing to a probably more realistic value for this model of $\gamma \simeq 0.49$. Since however this value is within the 1σ observational uncertainty, we will consider the originally fitted MGCDM γ value as the nominal one.

Using the above best fit γ values we present, in the lower panel of Fig. 3, the growth factor evolution derived by integrating Eq. (4.3) for the two cosmological models (MGCDM solid and Λ CDM dashed). The error bars correspond to the 1σ uncertainty of the fitted γ values. Note that the growth factors are normalized to unity at the present time. The difference between the fitted growth factors lies, at redshifts $z \geq 1$, in the interval ~ 1 –4%, while when using the theoretically predicted Λ CDM value of $\gamma_\Lambda \simeq 6/11$ the difference is less than 1.5%. For a consistent treatment of the two models and for the

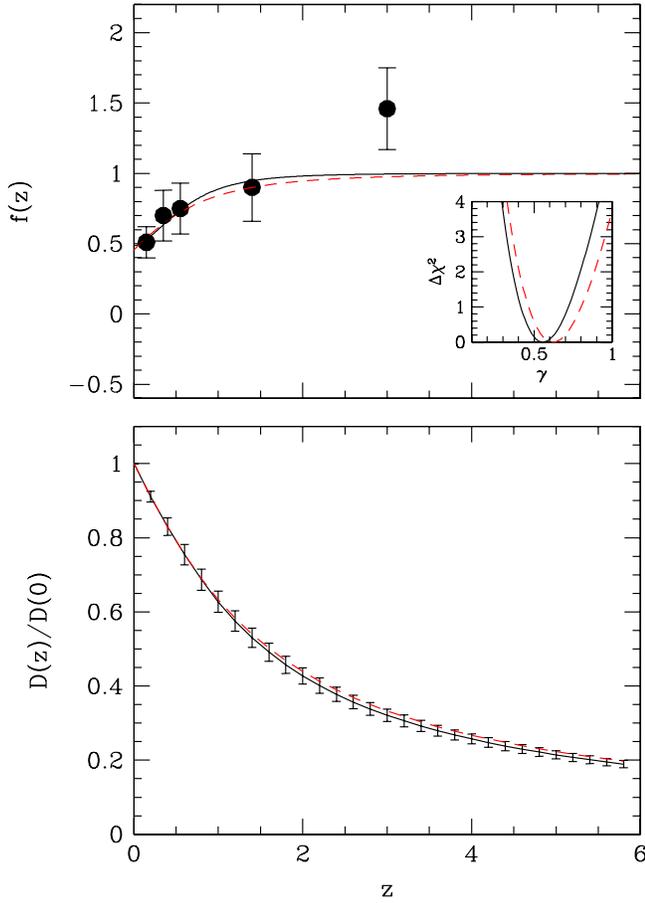


FIG. 3 (color online). *Upper Panel:* Comparison of the observed (solid circles [78], (see Table I) and theoretical evolution of the growth rate of clustering $f(z)$. The lines correspond to the MGCDM (solid curve) and the Λ CDM (dashed curve) models. *Bottom Panel:* The evolution of the growth factor, with that corresponding to the MGCDM model ($\gamma = 0.56$) showing a ~ 1 –4% difference with respect to that of the Λ CDM model ($\gamma_\Lambda = 0.62$), especially at large redshifts ($z \geq 1$). Error bars are plotted only for the MGCDM model in order to avoid confusion.

corresponding comparison of their respective mass functions and halo redshift distributions we will use, throughout the rest of the paper, the observationally derived γ values, i.e., $\gamma_\Lambda \simeq 0.62$ and $\gamma_{\text{MGCDM}} \simeq 0.56$.

V. COMPARE THE CLUSTER HALO ABUNDANCES

It is important to define observational criteria that will enable us to distinguish between the MGCDM model and the concordance Λ CDM cosmology. An obvious choice, that has been extensively used, is to compare the theoretically predicted cluster-size halo redshift distributions and to use observational cluster data to distinguish the models. Recently, the halo abundances predicted by a large variety of dark energy models have been compared with those corresponding to the Λ CDM model [16,55]. As a result,

such analyses suggest that many dark energy models explored in this study (including some of modified gravity) are clearly distinguishable from the Λ CDM cosmology.

We use the Press and Schechter [86] formalism, based on random Gaussian fields, which determines the fraction of matter that has formed bounded structures as a function of redshift. Mathematical details of our treatment can be found in [55]; here we only present the basic ideas. The number density of halos, $n(M, z)$, with masses within the range $(M, M + \delta M)$ are given by

$$n(M, z)dM = \frac{\bar{\rho}}{M} \frac{d \ln \sigma^{-1}}{dM} f_{\text{PSC}}(\sigma) dM, \quad (5.1)$$

where $f_{\text{PSC}}(\sigma) = \sqrt{2/\pi} (\delta_c/\sigma) \exp(-\delta_c^2/2\sigma^2)$, δ_c is the linearly extrapolated density threshold above which structures collapse [87], while $\sigma^2(M, z)$ is the mass variance of the smoothed linear density field, extrapolated to redshift z at which the halos are identified. It depends on the power-spectrum of density perturbations in Fourier space, $P(k)$, for which we use here the cold dark matter form according to [88], and the values of the baryon density parameter, the spectral slope and Hubble constant according to the recent WMAP7 results [5]. Although the Press-Schechter formalism was shown to provide a good first approximation to the halo mass function provided by numerical simulations, it was later found to over-predict/under-predict the number of low/high mass halos at the present epoch [89,90]. More recently, a large number of works have provided better fitting functions of $f(\sigma)$, some of them based on a phenomenological approach. In the present treatment, we adopt the one proposed by Reed *et al.* [91].

We remind the reader that it is traditional to parametrize the mass variance in terms of σ_8 , the rms mass fluctuations on scales of $8h^{-1}$ Mpc at redshift $z = 0$.

In order to compare the mass function predictions of the different cosmological models, it is imperative to use for each model the corresponding value of δ_c and σ_8 . It is well known that for the usual Λ cosmology $\delta_c \simeq 1.675$, while Weinberg and Kamionkowski [44] provide an accurate fitting formula to estimate δ_c for any dark energy model with a constant equation of state parameter. Since for the current graviton cosmological vacuum model the effective dark energy EoS parameter at the present time is $w \simeq 0$ which implies that the Hubble parameter is matter dominated it is fair to use the Einstein de-Sitter value $\delta_c \simeq 1.685$ [44]. Now, in order to estimate the correct model σ_8 power-spectrum normalization, we use the formulation developed in [55] which scales the observationally determined $\sigma_{8,\Lambda}$ value to that of any cosmological model. The corresponding MGCDM value is $\sigma_{8,\text{MGCDM}} = 0.828$ and it is based on $\sigma_{8,\Lambda} = 0.804$ (as indicated also in Table I), derived from an average of a variety of recent measurements (see also the corresponding discussion in [55]) which are based on the WMAP7 results [5], on a recent cluster abundances

analysis [92], on weak-lensing results [93] and on peculiar velocities based analyses [94].

Given the halo mass function from Eq. (5.1) we can now derive an observable quantity which is the redshift distribution of clusters, $\mathcal{N}(z)$, within some determined mass range, say $M_1 \leq M/h^{-1}M_\odot \leq M_2 = 10^{16}$. This can be estimated by integrating, in mass, the expected differential halo mass function, $n(M, z)$, according to

$$\mathcal{N}(z) = \frac{dV}{dz} \int_{M_1}^{M_2} n(M, z) dM, \quad (5.2)$$

where dV/dz is the comoving volume element. In order to derive observationally relevant cluster redshift distributions and therefore test the possibility of discriminating between the MGCDM and the Λ CDM cosmological models, we will use the expectations of two realistic future cluster surveys:

- the eROSITA satellite x-ray survey, with a flux limit of: $f_{\text{lim}} = 3.3 \times 10^{-14} \text{ erg s}^{-1} \text{ cm}^{-2}$, at the energy band 0.5–5 keV and covering $\sim 20\,000 \text{ deg}^2$ of the sky,
- the South Pole Telescope SZ survey, with a limiting flux density at $\nu_0 = 150 \text{ GHz}$ of $f_{\nu_0, \text{lim}} = 5 \text{ mJy}$ and a sky coverage of $\sim 4000 \text{ deg}^2$.

To realize the predictions of the first survey we use the relation between halo mass and bolometric x-ray luminosity, as a function of redshift, provided in [95], i.e.,

$$L(M, z) = 3.087 \times 10^{44} \left[\frac{ME(z)}{10^{15} h^{-1} M_\odot} \right]^{1.554} h^{-2} \text{ erg s}^{-1}. \quad (5.3)$$

The limiting halo mass that can be observed at redshift z is then found by inserting in the above equation the limiting luminosity, given by: $L = 4\pi d_L^2 f_{\text{lim}} c_b$, with d_L the luminosity distance corresponding to the redshift z and c_b the band correction, necessary to convert the bolometric luminosity of Eq. (5.3) to the 0.5–5 keV band of eROSITA. We estimate this correction by assuming a Raymond-Smith (1977) plasma model with a metallicity of $0.4Z_\odot$, a typical cluster temperature of $\sim 4 \text{ keV}$ and a Galactic absorption column density of $n_H = 10^{21} \text{ cm}^{-2}$.

The predictions of the second survey can be realized using again the relation between limiting flux and halo mass from [95]

$$f_{\nu_0, \text{lim}} = \frac{2.592 \times 10^8 \text{ mJy}}{d_A^2(z)} \left(\frac{M}{10^{15} h^{-1} M_\odot} \right)^{1.876} E^{2/3}(z), \quad (5.4)$$

where $d_A(z) \equiv d_L/(1+z)^2$ is the angular diameter distance out to redshift z .

In Fig. 4 (upper panels) we present the expected redshift distributions above a limiting halo mass, which is $M_1 \equiv M_{\text{limit}} = \max[10^{14} h^{-1} M_\odot, M_f]$, with M_f corresponding to the mass related to the flux limit at the different redshifts,

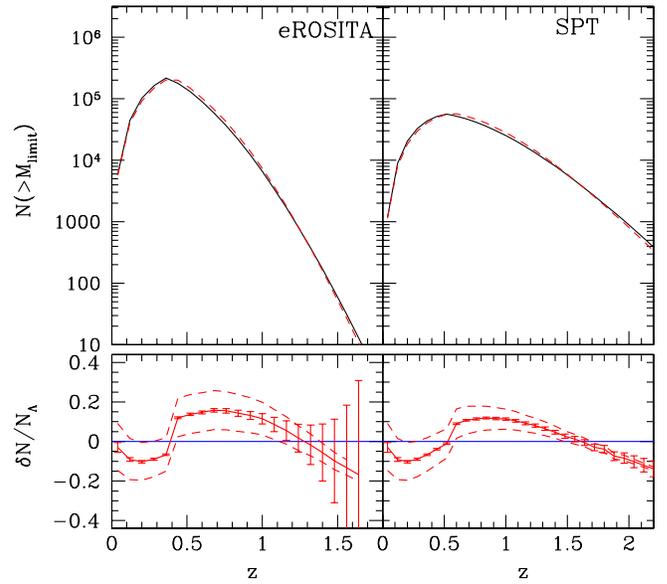


FIG. 4 (color online). The expected cluster redshift distribution of the MGCDM (solid curve) and Λ CDM (dashed curve) models for the case of two future cluster surveys (upper panels), and the corresponding fractional difference with respect to the reference Λ CDM model (lower panels). Error bars are 2σ Poisson uncertainties, while the dashed lines in the lower panel fitted the range due to the uncertainty of the observationally derived value of γ .

estimated by solving Eq. (5.3) and (5.4) for M . In the lower panels we present the fractional difference between the MGCDM and Λ CDM. The error bars shown correspond to 2σ Poisson uncertainties, which however do not include cosmic variance and possible observational systematic uncertainties, that would further increase the relevant variance. A further source of uncertainty that should be taken into account is related to the uncertainty of the observationally derived value of γ (see Sec. IV). The dashed lines in the lower panels of Fig. 3 bracket the corresponding number count relative model differences due to the 1σ uncertainty in the value of γ , with the lower curve corresponding to $(\gamma, \sigma_8) = (0.71, 0.787)$ and the upper to $(\gamma, \sigma_8) = (0.42, 0.876)$.

The results (see also Table II) indicate that significant model differences should be expected to be measured up to $z \lesssim 1$ for the case of the eROSITA x-ray survey, and to much higher redshifts for the case of the South Pole Telescope SZ survey. What is particularly interesting is the differential difference between the Λ CDM and MGCDM models, which is negative locally ($z \lesssim 0.3$), positive at intermediate redshifts ($0.4 \lesssim z \lesssim 1$) and negative again for $z \gtrsim 1$. This appears to be a unique signature of the MGCDM model, which differentiates it from the behavior of a large class of dark energy models (see [55]) and makes it relatively easier to be distinguished. In Table II, one may see a more compact presentation of our results including the relative fractional difference

TABLE II. Numerical results. The first column indicates the cosmological model. The second and third columns list the corresponding σ_8 and γ values, respectively. The remaining columns present the fractional relative difference of the abundance of halos between the MGCDM and the Λ CDM cosmology for two future cluster surveys discussed in the text. The lower two rows show results corresponding to the upper and lower 1σ range of the observational γ value uncertainty. Error bars are 2σ Poisson uncertainties and are shown only if they are larger than 10^{-2} .

Model	σ_8	γ	$(\delta\mathcal{N}/\mathcal{N}_\Lambda)_{\text{eROSITA}}$		$(\delta\mathcal{N}/\mathcal{N}_\Lambda)_{\text{SPT}}$		
			$z < 0.3$	$0.6 \leq z < 0.9$	$z < 0.3$	$0.6 \leq z < 0.9$	$1.3 \leq z < 2$
Λ CDM	0.804	0.62	0.00	0.00	0.00	0.00	0.00
MGCDM	0.831	0.56	-0.09	0.15 ± 0.01	-0.09	0.11	-0.03
MGCDM	0.875	0.42	0.00	0.25 ± 0.01	0.00	0.18	-0.01
MGCDM	0.789	0.71	-0.19	0.06 ± 0.01	-0.19	0.05	-0.05

between the MGCDM model and the Λ CDM model, in characteristic redshift bins and for both future surveys.

VI. CONCLUSIONS

In this work, the large and small scale dynamical properties of a flat FLRW cold dark matter cosmology, endowed with massive gravitons (MGCDM), were discussed from an analytical and a numerical viewpoints. We find that the MGCDM can accommodate a “dynamic phase transition” from an early decelerating phase (driven only by cold dark matter) to a late-time accelerating expansion and a subsequent recent reacceleration phase.

Interestingly, the Hubble function of the MGCDM model contains only two free parameters, namely H_0 and Ω_m , which is the same number of free parameters as the Λ CDM model. Performing, a joint likelihood analysis using the current observational data (SNIa, CMB shift parameter, and BAOs), we have provided tight constraints on the main cosmological parameter of the MGCDM model, i.e., $\Omega_m = 0.276 \pm 0.012$. We then compared the MGCDM scenario with the conventional flat Λ cosmology regarding the rate of clustering as well as the predicted halo redshift distribution.

The main conclusions of such a comparison are as follows:

- (i) At large redshifts the amplitude of the linear perturbation growth factor of the MGCDM model is slightly different to the Λ solution (at a 1–4% level), while the observationally determined growth index of clustering ($\gamma \simeq 0.56$) is smaller than the corresponding fit for the Λ model ($\gamma_\Lambda \simeq 0.62$), although within their respective 1σ uncertainties.
- (ii) The shape and amplitude for the redshift distribution of cluster-size halos predicted by the MGCDM model is quite different from the one of a flat Λ CDM cosmology. Such a difference depends on redshift and has a characteristic signature that can discriminate the current graviton model from other contender dark energy models in the future cluster surveys.

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