Dynamics and constraints of the dissipative Liouville cosmology

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ABSTRACT

In this article we investigate the properties of the FLRW flat cosmological models in which the cosmic expansion of the Universe is affected by a dilaton dark energy (Liouville scenario). In particular, we perform a detailed study of these models in the light of the latest cosmological data, which serves to illustrate the phenomenological viability of the new dark energy paradigm as a serious alternative to the traditional scalar field approaches. By performing a joint likelihood analysis of the recent supernovae type Ia data (SNIa), the differential ages of passively evolving galaxies, and the baryonic acoustic oscillations (BAOs) traced by the Sloan Digital Sky Survey (SDSS), we put tight constraints on the main cosmological parameters. Furthermore, we study the linear matter fluctuation field of the above Liouville cosmological models. In this framework, we compare the observed growth rate of clustering measured from the optical galaxies with those predicted by the current Liouville models. Performing various statistical tests we show that the Liouville cosmological model provides growth rates that match well with the observed growth rate. To further test the viability of the models under study, we use the Press–Schechter formalism to derive their expected redshift distribution of cluster-size halos that will be provided by future X-ray and Sunyaev–Zeldovich cluster surveys. We find that the Hubble flow differences between the Liouville and the LambdaCDM models provide a significantly different halo redshift distribution, suggesting that the models can be observationally distinguished.

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1. Introduction

The comprehensive study carried out in recent years by the cosmologists has converged towards a cosmic expansion history that involves a spatially flat geometry and a recent accelerating expansion of the Universe (see [1–8] and references therein). From a theoretical point of view, an easy way to explain this expansion is to consider an additional energy component, usually called dark energy (hereafter DE) with negative pressure, that dominates the Universe at late times. The simplest DE candidate corresponds to the cosmological constant (see [9–11] for reviews). Indeed the so-called spatially flat concordance ΛCDM model, which includes cold dark matter (CDM) and a cosmological constant (Λ), fits accurately the current observational data and thus it is an excellent candidate to be the model which describes the observed Universe.

Nevertheless, the identification of Λ with the quantum vacuum has brought another problem: the estimate of theoretical physicists that the vacuum energy density should be 120 orders of magnitude bigger than the measured Λ value. This is the "old" cosmological constant problem (CCP) [9]. The "new" problem [10] asks why is the vacuum density so similar to the matter density just now? Many solutions to both problems have been proposed in the literature [12–14]. An easy way to overpass the above theoretical problems is to replace the constant vacuum energy with a DE that evolves with time. Nowadays, the physics of DE is considered one of the most fundamental and challenging problems on the interface uniting Astronomy, Cosmology and Particle Physics and indeed in the last decade there have been theoretical debates among the cosmologists regarding the nature of this exotic component.

In the original scalar field models [15] and later in the quintessence context, one can ad hoc introduce an adjusting or tracker scalar field ϕ [16] (different from the usual SM Higgs field), rolling down the potential energy $V(\phi)$, which could resemble the DE [10,11,17–20]. This class of DE models have been widely used in the literature due to their simplicity. Notice that DE models with a canonical kinetic term have a dark energy EoS parameter $-1 \leqslant w_\phi < -1/3$. Models with ($w_\phi < -1$), sometimes called phantom DE [21], are endowed with a very exotic nature, like a scalar

field with negative kinetic energy. However, it was realized that the idea of a scalar field rolling down some suitable potential does not really solve the problem because ϕ has to be some high energy field of a grand unified theory (GUT), and this leads to an unnaturally small value of its mass, which is beyond all conceivable standards in particle physics. As an example, utilizing the simplest form for the potential of the scalar field, $V(\phi) = m_\phi^2 \phi^2/2$, the present value of the associated vacuum energy density is $\rho_\Lambda = \langle V(\phi) \rangle \sim 10^{-11}$ eV^4, so for $\langle \phi \rangle$ of order of a typical GUT scale near the Planck mass, $M_P \sim 10^{19}$ GeV, the corresponding mass of ϕ is expected in the ballpark of $m_\phi \sim H_0 \sim 10^{-33}$ eV.

Notice that the presence of such a tiny mass scale in scalar field models of DE is generally expected also on the basis of structure formation arguments [22–24]; namely from the fact that the DE perturbations seem to play an insignificant role in structure formation for scales well below the sound horizon. The main reason for this homogeneity of the DE is the flatness of the potential, which is necessary to produce a cosmic acceleration. Being the mass associated to the scalar field fluctuation proportional to the second derivative of the potential itself, it follows that m_ϕ will be very small and one expects that the magnitude of DE fluctuations induced by ϕ should be appreciable only on length scales of the order of the horizon. Thus, equating the spatial scale of these fluctuations to the Compton wavelength of ϕ (hence to the inverse of its mass) it follows once more that $m_\phi \lesssim H_0 \sim 10^{-33}$ eV.

Despite the above difficulties there is a class of viable models of quintessence usually called dilatonic that are based on supersymmetry, supergravity and string-theory and which can protect, for specific potentials, the light mass of quintessence (for a review see Ref. [25] and references therein). In particular, in string theory, gauge and gravitational couplings are connected with the vacuum expectation value of scalar field called dilaton ϕ [26]. In this context, it has been proposed by some of us a specific model for the DE. This DE model being associated with a rolling dilaton field that is a remnant of this non-equilibrium phase described by a generic non-critical (Liouville) string theory [27–30]. We call this scenario either Q-cosmology or Liouville cosmology. This pattern is based on non-critical (Liouville) strings [31–33], which offer a mathematically consistent way of incorporating time-dependent backgrounds in string theory. We note in passing that the presence of time dependent dilaton fields at late eras of the Universe, that characterizes Q-cosmology, may lead to phenomenologically interesting extensions of minimal supergravity models. The latter predict less dark-matter relic abundances than the conventional ΛCDM model, thereby allowing for more room for supersymmetry in collider (such as LHC) tests of the models [30,34].

In this article we investigate the main dynamical properties of the Q-cosmological model from the point of view of current astrophysical constraints, extending non-trivially earlier analyses [35,36]. Initially, a joint statistical analysis, involving the latest observational data (SNIa [4], $H(z)$ [37] and BAO [38–40]) is implemented. Secondly, we attempt to compare the matter fluctuation field of the Q-cosmology with that of the ΛCDM models by computing the growth rate of clustering. Finally, by using the Press–Schechter formalism [41] and recently derived functional forms for the dark matter halo mass function [42], we show that the evolution of the cluster-size halo abundances is a potential discriminator between the Q-cosmology and ΛCDM models. We would like to stress here that the abundance of collapsed structures, as a function of mass and redshift, can be accessed through observations [43]. Indeed, the mass function of galaxy clusters has been measured based on X-ray surveys [44–46], via weak and strong lensing studies [47–49], using optical surveys, like the SDSS [50,51], as well as, through Sunayev–Zeldovich (SZ) effect [52]. In the last decade many authors have found that the

abundance of collapsed structures is affected by the presence of a dark energy component [53–65].

The structure of our paper is as follows. The basic elements of the Liouville cosmological model are presented in Section 2, where we also introduce the cosmological equations for a flat Friedmann–Lemaitre–Robertson–Walker (FLRW) geometry. In Section 3, a joint statistical analysis based on SNIa, $H(z)$ and BAO is used to constraint the Liouville model free parameter. The issue related with the effective DE equation of state parameter and thus with the cosmic expansion is presented in Section 4. The linear growth factor of matter perturbations is discussed in Section 5, while in Section 6, we discuss and compare the corresponding theoretical predictions regarding the evolution of the cluster abundances. Finally, the main conclusions are summarized in Section 7.

2. Theoretical background on Q-cosmology

In non-critical Q-cosmological models the (effective, four-dimensional) Friedman equation is modified, including, apart from the standard matter and dark energy contributions denoted collectively by ρ, also Liouville-string off-shell corrections, $\Delta \rho$, which are not positive definite in general (without affecting, however, the overall positive-energy conditions of this non-equilibrium theory):

$$H^2(z) = \frac{8\pi G_N}{3} \rho + \Delta \rho \tag{1}$$

The detailed form of the system of dynamical equations, one of which is (1) is given by Ref. [66] in the Einstein frame [31], i.e. in the frame where the scalar curvature in the (off-shell) target space effective action assumes the canonical Einstein form to leading order in the Regge-slope α' expansion, are given by

$$3H^2 - \tilde{\varrho}_m - \varrho_\phi = \frac{e^{2\phi}}{2} \widetilde{\mathcal{G}}_\phi$$

$$2\dot{H} + \tilde{\varrho}_m + \varrho_\phi + \tilde{p}_m + p_\phi = \frac{\widetilde{\mathcal{G}}_{ii}}{a^2}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{1}{4}\frac{\partial \widehat{V}_{all}}{\partial \phi} + \frac{1}{2}(\tilde{\varrho}_m - 3\tilde{p}_m) = -\frac{3}{2}\frac{\widetilde{\mathcal{G}}_{ii}}{a^2} - \frac{e^{2\phi}}{2}\widetilde{\mathcal{G}}_\phi, \tag{2}$$

with

$$\widetilde{\mathcal{G}}_\phi = e^{-2\phi}(\ddot{\phi} - \dot{\phi}^2 + Qe^\phi \dot{\phi})$$

$$\widetilde{\mathcal{G}}_{ii} = 2a^2\left(\ddot{\phi} + 3H\dot{\phi} + \dot{\phi}^2 + (1-q)H^2 + Qe^\phi(\dot{\phi} + H)\right), \tag{3}$$

and q is the deceleration $q \equiv -\ddot{a}a/\dot{a}^2$. Above, an overdot denotes derivative with respect to the FLRW cosmic time, t, $\tilde{\rho}_m$, $\tilde{p}_m$ denote "matter" energy and pressure densities respectively, whilst

$$\varrho_\phi = \frac{1}{2}(2\dot{\phi}^2 + \widehat{V}_{all})$$

$$p_\phi = \frac{1}{2}(2\dot{\phi}^2 - \widehat{V}_{all}) \tag{4}$$

denote the dilaton energy density and pressure. In (4), the potential has the form $\widehat{V}_{all} = 2Q^2 \exp(2\phi) + V$, where $Q = Q(t)$ is the so-called central charge deficit function of the non-critical theory, which plays the role of a "gravitational friction" coefficient. The term V represents higher-string loop corrections in the target space-string effective action, which are in general not known in a closed form. In our work the precise form of V will not play a role, at least for late eras of the Universe, where such quantum string corrections are sub-dominant. The variation of the central charge deficit $Q(t)$ with the cosmic time t is provided by the following consistency equation, to leading order in the Regge-slope (α') expansion:

$$\frac{d\widetilde{\mathcal{G}}_\phi}{dt} = -6e^{-2\phi}(H + \dot{\phi})\frac{\widetilde{\mathcal{G}}_{ii}}{a^2}. \tag{5}$$

The reader should notice that, although we have assumed a (spatially) flat Universe, the terms on the r.h.s. of (2), which manifest departure from criticality, act in a sense like curvature terms as being non-zero at certain epochs.

The continuity equation for matter, which follows by combining the set of Eq. (2), reads:

$$\frac{d\tilde{\varrho}_m}{dt} + 3H(\tilde{\varrho}_m + \tilde{p}_m) + \frac{\dot{Q}}{2}\frac{\partial \hat{V}_{all}}{\partial Q} - \dot{\phi}(\tilde{\varrho}_m - 3\tilde{p}_m) = 6(H + \dot{\phi})\frac{\tilde{\mathcal{G}}_{ii}}{a^2}. \quad (6)$$

To proceed further one needs to make some extra assumptions. First we assume that the matter-energy density is split as

$$\tilde{\varrho}_m = \varrho_d + \varrho_r + \varrho_\delta. \quad (7)$$

The first term on the right-hand-side of (7) refers to as "dust", $w_d = 0$, and includes the baryonic matter and any other sort of matter, characterized by $w = 0$, which does not feel the effect of the non-critical terms. Following Ref. [66] we may assume that dust does not couple to the dilaton and the non-critical-stringy corrections at late eras, so that its respective density, ϱ_d, obeys the standard energy conservation equation (c.f. (6)):

$$\frac{d\varrho_d}{dt} + 3H\varrho_d \simeq 0. \quad (8)$$

The second term in (7) refers to radiation $w_r = \frac{1}{3}$ and the third term to an unknown sort of *exotic* matter which is characterized by an equation of state (hereafter EoS) having a weight w_δ. In our analysis for late epochs of the Universe, such as the eras of structure formation, we may assume that this sort of matter which would otherwise feel the non-critical terms in the evolution of the Universe, feels predominantly only the time-dependent dilaton $\dot{\phi}$ terms in (6), i.e. its density, ϱ_δ, obeys approximately the following energy conservation equation:

$$\frac{d\varrho_\delta}{dt} + 3H\left(1 - \frac{\dot{\phi}}{3H} + w_\delta(z)\left(1 + \frac{\dot{\phi}}{H}\right)\right)\varrho_\delta \simeq 0, \quad (9)$$

for late eras of the Universe, e.g. corresponding to redshifts $z \leqslant 2$, where analytic treatments of the Liouville Q-cosmology are available. In general the exotic matter equation of state $w_\delta(z)$ depends on the redshift, but in our analysis in this work we consider it to be a constant.

For late epochs of the Universe, an approximate scaling behavior of the dilaton field may be found by making the further assumption that its configuration is not affected much by the presence of matter. In such a case, one may use the purely gravitational solution of Ref. [28], which expresses the dilaton in the form[1]:

$$\phi \simeq c_1 + \phi_0 \ln(a(t)), \quad (10)$$

in units of the present scale factor $a_0 = 1$, where ϕ_0, c_1 are constants. In the purely gravitational theory of [28], $\phi_0 = -1$, but here we want to be more general, in order to account for the presence of matter, and so we shall consider the constant ϕ_0 as a phenomenological parameter to be determined by fitting the data. However, we shall assume $\phi_0 < 0$, which assumes a perturbatively weak string coupling $g_s = e^\phi$ as the cosmic time elapses, which is crucial for the validity of our analytical treatment at late epochs of the Universe. In particular, this implies that string loop corrections, although non negligible, and in fact their inclusion may even lead to some interesting physical effects (see below) [35], nevertheless they can be treated perturbatively, and thus can lead to analytic solutions at late eras.

With the above in mind, we thus observe that from (10) and (9) we obtain:

$$\frac{d\varrho_\delta}{dt} + 3H\left(1 - \frac{\phi_0}{3} + w_\delta(1 + \phi_0)\right)\varrho_\delta \simeq 0, \quad (11)$$

implying the following scaling behavior of the exotic matter with the scale factor:

$$\varrho_\delta \sim a^{-\delta}, \quad \delta = 3\left(1 - \frac{\phi_0}{3} + w_\delta(1 + \phi_0)\right). \quad (12)$$

In particular, it has been found that $\delta \sim 4$ [36] can fit the BAO data [38]. Such a value is in the ballpark of theoretically expected values. For instance, the simplest phenomenological assumption on the pertinent EoS parameter $w_\delta = \frac{p_\delta}{\varrho_\delta}$ of the exotic dark matter fluid, consistent with Big-Bang-nucleosynthesis constraints, is that $w_\delta \sim 0.4$ [66]. To obtain $\delta \sim 4$, while maintaining the feature $\phi_0 < 0$, one must use either values of $|\phi_0| \ll 3$, implying a weak dependence of the dilaton on the scale factor, or $\phi_0 \sim -1$ and more generally the condition

$$\frac{\dot{\phi}}{H} \sim -1,$$

in which case (9), (11) become independent of w_δ.

On the other hand, the BBN constraint is also satisfied in models where the dilaton dominates the early eras of the Universe [34], in the absence of non-critical string effects, with the exotic matter behaving as dust $w_\delta = 0$ coupled to the dilaton. In this case, the scaling exponent (12) becomes: $\delta \simeq 3 + |\phi_0|$. The phenomenologically likely [36] value $\delta \sim 4$ is then achieved for $|\phi_0| \sim 1$, which is in the range of the analytical solutions for late eras found in [28], and for which, as already mentioned, the energy Eq. (11) is largely independent of w_δ.

Following the above discussion, we therefore parametrize the normalized Hubble parameter of this model, $E(z) = H(z)/H_0$, where z is the redshift, as follows [35]:

$$E(z) = [\Omega_3(1 + z)^3 + \Omega_\delta(1 + z)^\delta + \Omega_2(1 + z)^2]^{1/2} \quad (13)$$

with

$$\Omega_3 + \Omega_\delta + \Omega_2 = 1. \quad (14)$$

It is interesting to mention that the current normalized Hubble function Eq. (13) has a form which is close to a polynomial one. Note that a quadratic polynomial dark energy has been proposed by Sahni et al. [67]. We would like to stress once again that the above formulas are valid for late eras, such as the ones pertinent to the supernova and other data ($0 \leqslant z \leqslant 2$).

In this framework, the various Ω_i contain contributions from *both* dark energy and matter energy densities. Specifically, Ω_3 does not merely represent ordinary matter effects, but also receives contributions from the dilaton dark energy. In fact, the sign of Ω_3 depends on details of the underlying theory, and it could even be *negative*. In a similar vein, the exotic contributions scaling as $(1 + z)^\delta$ are affected by the off-shell Liouville terms of Q-cosmology. It is because of the similar *scaling behaviors* of dark matter and dilaton dark energy that we reverted to the notation Ω_i, $i = 2, 3, \delta$ in Refs. [35,36]. In order to distinguish ordinary matter from dilaton dark energy effects that scale similarly with the redshift, one would have to perform also measurements of the effective dark energy EoS parameter (see Section 4).

3. Likelihood analysis

Let us now discuss the statistical treatment of the observational data used to constrain the Liouville model presented in the previous section. To begin with, we consider the *Union* supernovae Ia set

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[1] A similar behavior of the dilaton is also valid in the early Universe as discussed in [34]. Such forms depend crucially on the form of the dilaton potential, which should be of exponential form $V \sim e^{-c\phi}$, c some constant. Dilaton Q-cosmologies are indeed characterized by such potentials [66,35].

of Kowalski et al. [4] which contains 307 entries. The likelihood estimator is determined by a χ^2_{SNIa} statistics:

$$\chi^2_{SNIa} = \sum_{i=1}^{307} \left[\frac{\mu_{th}(z_i, \Omega_3, \Omega_\delta) - \mu_{obs}(z_i)}{\sigma_i} \right]^2, \qquad (15)$$

where z_i is the observed redshift, μ is the distance modulus $\mu = m - M = 5\log d_L + 25$ and d_L is the luminosity distance,

$$d_L(z) = (1 + z)r(z) \quad r(z) = c \int_0^z \frac{dy}{H(y)}. \qquad (16)$$

In addition to the SNIa data, we also consider the BAO scale produced in the last scattering surface by the competition between the pressure of the coupled baryon-photon fluid and gravity. The resulting acoustic waves leave (in the course of the evolution) an overdensity signature at certain length scales of the matter distribution. Evidence of this excess was recently found in the clustering properties of SDSS galaxies [38–40] and it provides a suitable "standard ruler" for constraining dark energy models. We would like to remind the reader that in the Liouville cosmology the ordinary matter component which appears in the normalized Hubble parameter Eq. (13) is mixed with the "exotic" contributions. Thus we can not use the measurements of BAO derived by [38–40]. To overcome this problem Mavromatos & Mitsou [36] have proposed a different estimator which is valid for Liouville cosmologies. This is

$$B \equiv \left[\left(\int_0^{z_{BAO}} \frac{dz}{E(z)} \right)^2 \frac{z_{BAO}}{E(z_{BAO})} \right]^{1/3} \qquad (17)$$

where $z_{BAO} = 0.35$. Note that the measured value is $B = 0.334 \pm 0.021$ [36]. Therefore, the corresponding χ^2_{BAO} function can be written as:

$$\chi^2_{BAO} = \frac{[B(\Omega_3, \Omega_\delta) - 0.334]^2}{0.021^2}. \qquad (18)$$

Finally, a very interesting geometrical probe of dark energy is provided by the measures of $H(z)$ [37] from the differential ages of passively evolving galaxies (hereafter $H(z)$ data). Note that the sample contains 11 entries spanning a redshift range of $0 \leqslant z < 2$. In this case the corresponding χ^2_H function can be written as:

$$\chi^2_H = \sum_{i=1}^{11} \left[\frac{H_{th}(z_i, \Omega_3, \Omega_\delta) - H_{obs}(z_i)}{\sigma_i} \right]^2, \qquad (19)$$

where $H(z)$ is the Hubble parameter,[2] $H(z) = H_0 E(z)$.

In order to put tighter constraints on the corresponding parameter space of our cosmological model, the above probes are combined through a joint likelihood analysis,[3] given by the product of the individual likelihoods according to: $\mathcal{L}_{tot} = \mathcal{L}_{SNIa} \times \mathcal{L}_{BAO} \times \mathcal{L}_H$, which translates in the joint χ^2 function in an addition: $\chi^2_{tot} = \chi^2_{SNIa} + \chi^2_{BAO} + \chi^2_H$. The resulting best fit parameters for different values of δ, are presented in Table 1. The current statistical results are in very good agreement with those found by [36].

The reader may worry that negative dust-like energy densities indicate an instability of the model and an obvious violation of the positive energy conditions, and thus its immediate exclusion. However, as discussed in [35,36], and already mentioned in the previous section, the "dust"-like contributions, Ω_3, do not merely represent ordinary matter effects, but also receive contributions from the dilaton dark energy. In fact, the sign of Ω_3 depends on details of the underlying theory, and it could even be negative. For instance, as argued in [35], string loop corrections could lead

to a negative Ω_3. In addition, Kaluza–Klein graviton modes in certain brane-inspired models [69] also yield negative dust contributions. In a similar vein, the exotic contributions scaling as $(1 + z)^\delta$ are affected by the off-shell Liouville terms of Q-cosmology.

4. The recent expansion history

In Fig. 1 we plot the Hubble parameter of the current Liouville models as a function of redshift, which for redshifts $z \gtrsim 1.5$ appears to be different in amplitude with respect to the corresponding ΛCDM model expectations (dot-dashed line).

As we have already mentioned in Section 2 the density related with the Ω_3 parameter varies as $\varrho_3(z) \propto (1 + z)^3$. Owing to the fact that the density of ordinary matter (baryons and usual cold dark matter), denoted ϱ_m from now on, follows the same power-law as $\varrho_3(z)$, $\varrho_m(z) \propto (1 + z)^3$, we are completely justified to assume that the usual matter density is one of the components of the density $\varrho_3(z)$. In other words the $\Omega_m(z) = \varrho_m(z)/\varrho_c(z)$ density parameter (where $\varrho_c(z) = 3H^2(z)/8\pi G$ is the critical density parameter) is included in the $\Omega_3(z)$ parameter. On the other hand, as a result of the existence of non-trivial dilaton couplings to some dark matter species, as mentioned previously, there is dark matter with exotic scaling $\Omega_\delta(1 + z)^\delta$, with δ given by (12). Although this is *not* a dark energy contribution, nevertheless it is the dilaton couplings that modify its scaling, and therefore in this sense, the origin of this exotic scaling may be attributed to the existence of the interaction of dilaton dark energy with dark matter. However, since the $\Omega_\delta > 0$ represents massive matter, it may cluster and lead to non trivial contributions to structure growth in the Universe. In this sense, it is distinguishable from dark energy contributions that are non clustering. One of our points in this work is an attempt to distinguish phenomenologically whether the current astrophysical data

Table 1
Results of the likelihood function analysis. The 1st column indicates the Liouville model. 2nd column presents the values of δ used. 3rd, 4th and 5th columns show the best fit parameters and the reduced χ^2_{tot}. In the final column one can find various line types of the models appearing in Figs. 1 and 2.

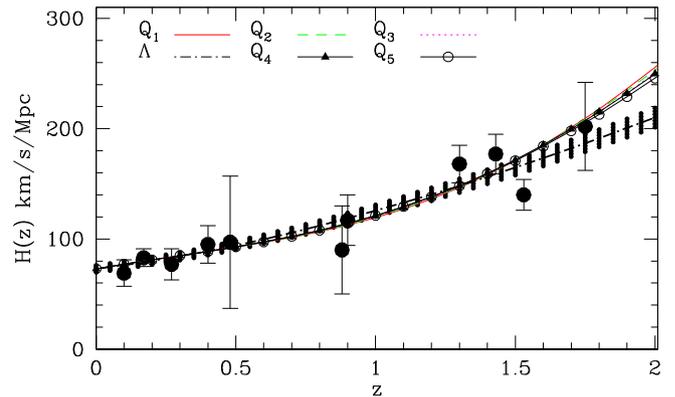| Model	δ	Ω_3	Ω_δ	$\chi^2_{tot}/317$	Symbols
Q_1	4.3	-1.43 ± 0.14	0.27 ± 0.02	1.030	Solid
Q_2	4.1	-1.64 ± 0.16	0.40 ± 0.04	1.028	Dashed
Q_3	3.9	-1.93 ± 0.19	0.59 ± 0.06	1.027	Dotted
Q_4	3.7	-2.45 ± 0.25	0.95 ± 0.10	1.025	Triangles
Q_5	3.5	-3.29 ± 0.33	1.63 ± 0.17	1.024	Open circles

Fig. 1. Comparison of the observed (solid circles [37]) and theoretical evolution of the Hubble parameter, $H(z)$, using $H_0 = 73.8 \pm 2.4$ km/s/Mpc [68]. The different Liouville models are represented by different symbols and line types (see Table 1 for definitions). For comparison, the dot-dashed line corresponds to the traditional ΛCDM model, while the thick-line error bars correspond to 1σ H_0-uncertainties. We do not plot the 1σ H_0-uncertainties for the Liouville models in order to avoid confusion.

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[2] The Hubble constant is $H_0 = 73.8 \pm 2.4$ km/s/Mpc [68].

[3] Likelihoods are normalized to their maximum values. In the present analysis we always report 1σ uncertainties on the fitted parameters. Note also that the total number of data points used here is $N_{tot} = 319$, while the associated degrees of freedom are: $dof = 317$. Note that we sample $\Omega_3 \in [-4, 1]$ and $\Omega_\delta \in [0.1, 2]$ in steps of 0.001.

point out towards clustering of this exotic matter, and hence contribution to structure formation and growth, or not.

To this end, we first write the normalized Hubble parameter Eq. (13) as follows

$$E(z) = \frac{H(z)}{H_0} = [\Omega_m(1+z)^3 + \Delta H(z)^2]^{1/2} \tag{20}$$

with

$$\Delta H(z)^2 = \Omega_\delta(1+z)^\delta + (\Omega_3 - \Omega_m)(1+z)^3 + \Omega_2(1+z)^2. \tag{21}$$

Notice here that in the above formula we have been careful to isolate the matter contributions, pertaining to dust Ω_d (for baryons and dust dark matter $\Omega_i = \varrho_{i0}/\varrho_{c0}$), from the exotic matter terms, Ω_δ with non-trivial equation of state w_δ, corresponding to a scaling exponent δ (12). It should be stressed that the last term $\Delta H^2(z)$ of the normalized Hubble function (20) encode the Q-cosmology corrections to the standard FLRW expression.

Since the corresponding components exhibit smooth scaling with the redshift, the absorption of these extra effects to an "effective dark energy" contribution, with a non-trivial EoS is possible. In general, it is well known that one can express the effective dark energy EoS parameter in terms of the normalized Hubble parameter [70]

$$w_{DE}(a) = \frac{-1 - \frac{2}{3}\frac{d\ln E}{d\ln a}}{1 - \Omega_m(a)}, \tag{22}$$

where in our case

$$\Omega_m(a) = \frac{\Omega_m a^{-3}}{E^2(a)}. \tag{23}$$

Differentiating the latter and utilizing Eq. (22) we find that

$$\frac{d\Omega_m}{d\ln a} = 3w_{DE}(a)\Omega_m(a)[1 - \Omega_m(a)]. \tag{24}$$

Notice that we will use the above quantity in Section 5.

After some simple algebra, it is also readily seen that the effective dark energy EoS parameter is given by (see [71]):

$$w_{DE}(a) = -1 - \frac{1}{3}\frac{d\ln \Delta H^2}{d\ln a} \tag{25}$$

or

$$w_{DE}(z) = -1 + \frac{1}{3}\frac{d\ln \Delta H^2}{d\ln(1+z)} \tag{26}$$

where $a = (1+z)^{-1}$. From the above analysis it becomes clear that any modifications to the EoS parameter are included in the second term of Eq. (26). A similar analysis for other DE models can be found in [71,72].

In our case, on inserting Eq. (21) into Eq. (26) it is straightforward to obtain a simple analytical expression for the effective dark energy EoS parameter:

$$w_{DE}(z) = -1 + \frac{1}{3}\frac{3(\Omega_3 - \Omega_m)(1+z) + \delta\Omega_\delta(1+z)^{\delta-2} + 2\Omega_2}{(\Omega_3 - \Omega_m)(1+z) + \Omega_\delta(1+z)^{\delta-2} + \Omega_2}. \tag{27}$$

It thus follows that in the current cosmological context, the Liouville scenario as proposed by Ellis et al. [35] can be treated by an additional effective fluid with EoS parameter defined by Eq. (27). In order to investigate the behavior of the effective EoS parameter we have to know a priori the value of Ω_m. The matter density Ω_m remains the most weakly constrained cosmological parameter. In principle, Ω_m is constrained by the maximum likelihood fit to the WMAP [6] and SNIa [4] data in the context of the concordance ΛCDM cosmology. However, in the spirit of this work, we want to use measures which are completely independent of the dark energy

component. An estimate of Ω_m without conventional priors is not an easy task in observational cosmology. Nevertheless, various authors, using mainly large scale structure studies, have attempted to set constraints on the Ω_m parameter. In a rather old paper, Plionis et al. [73], using the motion of the Local Group with respect to the cosmic microwave background, found that $\Omega_m \simeq 0.30$. From the analysis of the power spectrum, Sanchez et al. [74] obtained a value $\Omega_m \simeq 0.24$. Moreover, the authors of Refs. [75,76] analyzed the peculiar velocity field in the local Universe and obtained the values $\Omega_m \simeq 0.30$ and $\simeq 0.22$ respectively. In addition, the authors of Ref. [77], based on the cluster mass-to-light ratio, claim that Ω_m lies in the interval $0.15 - 0.26$ (see also [78] for a review). Therefore, there are strong independent indications that Ω_m lies in the range $0.2 \lesssim \Omega_m \lesssim 0.3$. In order to compare our results with those of the flat ΛCDM model, we shall restrict our present analysis to an indicative value of $\Omega_m = 0.28$.

In order to visualize the redshift dependence of the effective EoS parameter, we compare in the upper panel of Fig. 2 various flat Liouville cosmological models (see Table 1). One can divide the evolution of the cosmic expansion history in different phases on the basis of the varying behavior of the Liouville and ΛCDM models. We will investigate such variations in terms of the deceleration parameter, $q(z) = -[1 - d\ln H/d\ln(1+z)]$, which is plotted in the lower panel of Fig. 2. We can divide the cosmic expansion history in the following phases:

- At early enough times $z \sim 2$ the deceleration parameters of both models are positive with $q_Q < q_\Lambda$, which means that the ΛCDM model is more decelerating than the Liouville model.
- For $1.6 \lesssim z < 1.9$ the deceleration parameters are both positive with $q_Q > q_\Lambda$, which means that the cosmic expansion
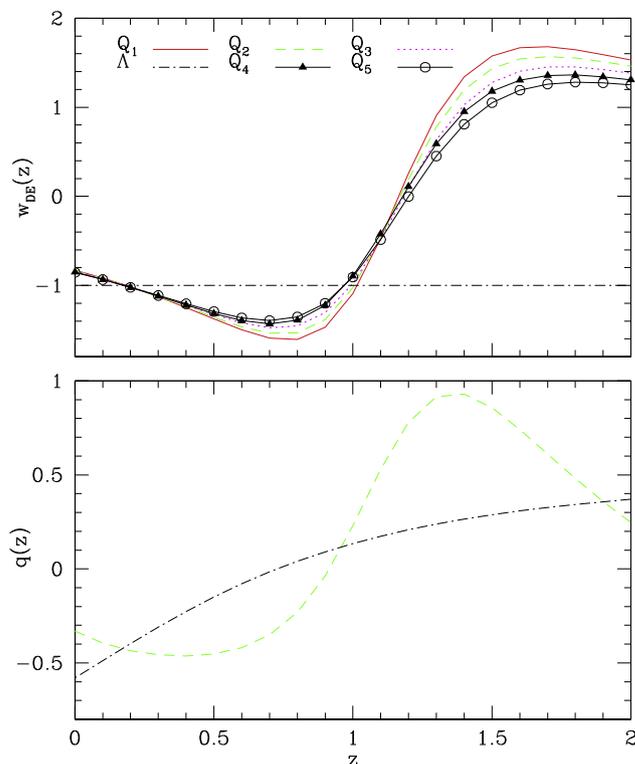
Fig. 2. Expansion history. In the upper panel we display the evolution of the dark energy effective EoS parameter. The different Liouville models are represented by different symbols and line types (see Table 1 for definitions). In the lower panel we compare the deceleration parameters of the Q_2 and the concordance ΛCDM (dot-dashed line) models. The deceleration parameter is plotted only for the Q_2 model in order to avoid confusion.

in the Liouville model is more rapidly "decelerating" than in the ΛCDM case. Note that the effective EoS parameter of the Q-model tends to its maximum value, $w_{DE}(z) \sim 1.4$, while we always have $w_{DE} = -1$ for the ΛCDM model (dot-dashed line);

- Between $1.1 \leqslant z < 1.6$ the deceleration parameters remain positive (with $q_Q < q_\Lambda$) but q_Q decreases rapidly.
- For $0.7 < z < 1.1$ the traditional ΛCDM model remains in the decelerated regime ($q_\Lambda > 0$) but the Liouville model is starting to accelerate ($q_Q < 0, w_{DE} < -1/3$). At $z \simeq 0.9$ the corresponding effective EoS parameter cross, for the first time, the phantom divide ($w_{DE} = -1$) and it stays there for some time ($0.2 < z < 0.9$).
- Prior to the present epoch the effective EoS parameter crosses the phantom divide for the second time ($z \simeq 0.2$) and then it remains close to $w_{DE} \simeq -0.90$. The deceleration parameters are both negative and since $q_Q > q_\Lambda$, the ΛCDM model provides a stronger acceleration than in the Liouville model. At the present time we find $q_{Q,0} \simeq -0.33$ and $q_{\Lambda,0} \simeq -0.60$

5. The growth factor and the rate of clustering

In this section, we discuss the basic equation which governs the behavior of the matter perturbations on sub-horizon scales and within the framework of any dark energy model, including those of modified gravity ("geometrical dark energy").

The basic assumption underlying the approach is that whatever we call "dark energy" contribution in our effective approach, pertains to a substance that is non clustering to participate significantly in the growth of cosmic structures. In our Liouville cosmology, this issue is subtle. As we have discussed above, we do have two kinds of matter in the model, with two different equations of state and scaling, an ordinary dust, and an exotic dark matter component, with non trivial scaling exponent δ (12), as a result of its coupling to the dilaton. The phenomenologically likely value $\delta \sim 4$ makes the behavior of exotic dark matter resembling (to a good approximation) that of relativistic matter (radiation), which does not cluster. We shall come back to this point later one. For the moment let us restrict ourselves to the ordinary matter as the dominant kind that contributes to structure formation via its clustering properties.

For these cases, a full analytical description can be introduced by considering an extended Poisson equation together with the Euler and continuity equations. Consequently, we consider here the evolution equation of the matter fluctuations for models where the DE fluid has a vanishing anisotropic stress and it is not coupled to ordinary matter (noninteracting DE see [79,72,80–84]). It is well known that for small scales (smaller than the horizon) the dark energy component (or "geometrical" dark energy) is expected to be smooth and thus it is fair to consider perturbations only on the usual matter component of the cosmic fluid [85]. This assumption leads to the traditional equation for matter perturbations (see Appendix A):

$$\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G\rho_m\delta_m,\tag{28}$$

where ρ_m is the matter density which clusters in order to form cosmic structures.

We would like to emphasize here that Eq. (28) can be used also in the case of a dilaton field, as it has been shown by Boisseau et al. [86][4]

[4] The authors of [86] have worked in the so-called Jordan frame, in which the dilaton couples to the Einstein sacalar curvature term in the (four space–time dimensional) action through a function $F(\phi)$:

$$S = \int d^4x \frac{F(\phi)}{2} R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_{\text{dust matter}}(g_{\mu\nu}).$$

In this formalism there is an effective gravitational "constant", $G_{\text{eff}}(\phi)$, which is a function of the dilaton that replaces G in (28). Notice that the matter action does not couple to the dilaton in the Jordan frame. In our Liouville string cosmology case we have an additional component to the matter action, $\mathcal{L}_{\text{exotic matter}}(g_{\mu\nu}, \phi)$, involving the exotic dark matter, with equation of state w_δ, and its interactions with the ordinary baryonic or dark matter with dust scaling. This depends on the dilaton in the Jordan frame. In the Liouville Cosmology we work in the Einstein frame, as already mentioned, by having redefined appropriately the gravitational field, in such a way that, in terms of the new metric field, there is a canonically normalized Einstein term in the effective action. Such a redefinition leaves the perturbative scattering amplitudes of low-energy string theory invariant, and hence is an allowed transformation in the string framework that does not affect the physical predictions. Under this redefinition, the matter action involves coupling of the dilaton with the dust-like matter fields, as a consequence of the redefined gravition. Nevertheless, we still assume for our purposes that such a coupling is very weak, and the only dominant couplings of the dilaton to matter fields in the Einstein frame are the ones pertaining to the exotic dark matter components. As we shall discuss below, such assumptions can be put directly to experimental tests by studying galactic growth data. In other words, Eq. (28) is valid also in the Liouville cosmology, as far as ordinary dust matter (including potential dark matter components scaling like dust) is concerned.

Within this latter framework, we assume that for small scales (smaller than the horizon) the effective dark energy component $\Delta H(z)$ appeared in the modified Friedmann Eq. (20) is smooth (homogeneous) which implies that Eq. (28) is valid also in the Liouville model. If we change the variables from t to $\ln a$ ($\frac{d}{dt} = H\frac{d}{d\ln a}$) then the time evolution of the mass density contrast (see Eq. (28)) takes the following form [87]

$$(\ln \delta_m)'' + (\ln \delta_m)'^2 + (\ln \delta_m)'X(a) = \frac{3}{2}\Omega_m(a),\tag{29}$$

where

$$X(a) = \frac{1}{2} - \frac{3}{2}w_{DE}(a)[1 - \Omega_m(a)].\tag{30}$$

The prime denotes derivatives with respect to $\ln a$. Now, for any type of dark energy an efficient parametrization of the matter perturbations is based on the growth rate of clustering [88]

$$f(a) = \frac{d \ln \delta_m}{d \ln a} = \Omega_m^\gamma(a)\tag{31}$$

γ is the so called growth index [87,71,79,82,89] which plays a key role in cosmological studies, especially in light of recent large redshift surveys, such as the *WiggleZ* [90,91]. Indeed, it has been proposed that measurement of the growth index can provide an efficient way to discriminate between modified gravity models and DE models which adhere to general relativity. As an example, it was theoretically shown that for DE models within the general relativity framework, the growth index γ is well fitted by $\gamma_{GR} \approx 6/11$ [82,89]. Notice, that in the case of the braneworld model of Ref. [95] we have $\gamma \approx 11/16$ (see also [82]), while for the $f(R)$ gravity models we have $\gamma(z) \approx 0.41$ at $z = 0$ [84].

Now, inserting Eq. (31) into Eq. (29) and using simultaneously Eq. (24) we obtain (see also [87])

$$3w_{DE}(a)\Omega_m(a)[1 - \Omega_m(a)]\frac{df}{d\Omega_m} + f^2 + fX(a) = \frac{3}{2}\Omega_m(a)\tag{32}$$

Table 2
Data of the growth rate of clustering. The correspondence of the columns is as follows: redshift, observed growth rate, cosmological parameters use by different authors and references.

z	$f_{obs,Ref}$	$(\Omega_{m,Ref}, \sigma_{8,Ref})$	Refs.
0.15	0.49 ± 0.14	(0.30, 0.90)	[99–101]
0.22	0.60 ± 0.10	(0.27, 0.80)	[90]
0.35	0.70 ± 0.18	(0.24, 0.76)	[102]
0.41	0.70 ± 0.07	(0.27, 0.80)	[90]
0.55	0.75 ± 0.18	(0.30, 1.00)	[103]
0.60	0.73 ± 0.07	(0.27, 0.80)	[90]
0.77	0.91 ± 0.36	(0.27, 0.78)	[99]
0.78	0.70 ± 0.08	(0.27, 0.80)	[90]
1.40	0.90 ± 0.24	(0.25, 0.84)	[104]
2.42	0.74 ± 0.24	(0.26, 0.93)	[105]
3.00	1.46 ± 0.29	(0.30, 0.85)	[98]

In this framework, Nesseris & Perivolaropoulos [89] used for the first time growth-rate data (most of the data are included in our Table 2) in order to constraint γ. They found (see also [87,82]) that a good approximation of the growth index, especially at $z \sim 1$, is given by:

$$\gamma \simeq \frac{3(w_{DE} - 1)}{6w_{DE} - 5}. \tag{33}$$

Of course the above formula boils down to 6/11 for the usual ΛCDM [$w_{DE}(z) = -1$] model as it should.[5]

Since the effective EoS parameter of the Liouville cosmological model varies strongly with redshift (see Fig. 2) one would expect that the corresponding growth index would be a function of redshift. Following the notations of Polarski and Gannouji [96] we consider a first order expansion of γ in redshift

$$\gamma(z) = \gamma_0 + \gamma_1 z, \tag{34}$$

where γ_0 is given by Eq. (33). Using Eqs. (34) in (31) and (32) we obtain (see also [96])

$$\gamma_1 = \frac{\Omega_m^{\gamma_0} + 3(\gamma_0 - \frac{1}{2})w_{DE,0}(1 - \Omega_m) - \frac{3}{2}\Omega_m^{1-\gamma_0} + \frac{1}{2}}{\ln \Omega_m}, \tag{35}$$

where γ_0 is given by Eq. (33). We would like to remind the reader that in the current article we use $w_{DE,0} \simeq -0.90$ and $\Omega_m \simeq 0.28$ which imply that $(\gamma_0, \gamma_1) \simeq (0.550, -0.048)$. We also find that the latter set of values remains quite robust within the range of the physically accepted values of Ω_m. Indeed as an example in the case of $\Omega_m = 0.30$ we find $(\gamma_0, \gamma_1) \simeq (0.549, -0.046)$.

In Table 2, we quote the observational values of the growth rate of clustering, with the corresponding error bars. This is an enriched version of the dataset used in [89], which includes in addition data from the *WiggleZ* [90]. The observed growth rate of clustering is given by $f_{obs} = \beta b$, where $\beta(z)$ is the redshift space distortion parameter and $b(z)$ is the linear bias. Observationally, one can estimate the $\beta(z)$ parameter by using the anisotropy of the correlation function. The linear bias factor can be defined as the ratio of the variances of the tracer (galaxies, QSOs, etc.) and underlying mass density fields, smoothed out at the scale $8h^{-1}$ Mpc, at which the variance is of order unity: $b(z) = \sigma_{8,tr}(z)/\sigma_8(z)$, where $\sigma_{8,tr}(z)$ is measured directly from the sample. As discussed in [89], the weak point of the $f_{obs}(z)$ data is the fact that the $\sigma_8(z)$ is defined at each redshift using a particular reference (fiducial) ΛCDM model (see third column in Table 2). Moreover, the growth measurements

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[5] We should point out at this stage that in order to compare the observed growth history of the Universe with predictions coming from modified gravity models, which however is not our case, one cannot use the observed $f_{obs}(z)$ but rather a combination of the growth rate of structure and the rms fluctuations of the linear density field on scales of $8h^{-1}$ Mpc, namely $f(z)\sigma_8(z)$. This is a model independent quantity (see e.g. [91] and references therein).
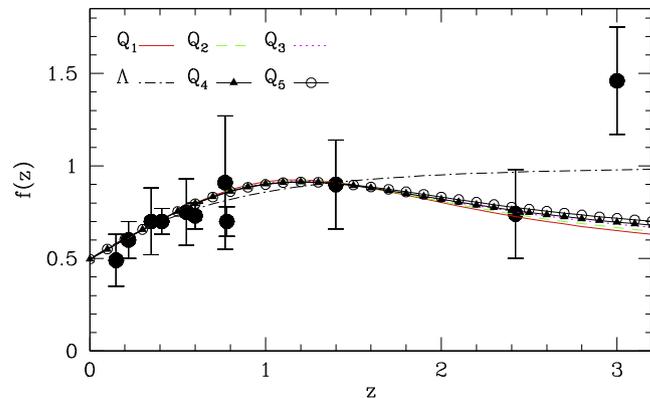
Fig. 3. Comparison of the observed, (solid circles; see Table 2), and theoretical evolution of the growth rate of clustering $f(z)$. Note that the different Liouville models are represented by different symbols and line types (see Table 1 for definitions). The dashed line corresponds to the ΛCDM cosmology.

are correlated with the shape of the power spectrum, which makes the growth rate sample rather heterogeneous [91,92]. In our analysis, we ignore the effects of such a heterogeneity. Such simplifying assumptions in treating the growth rate data have been used extensively in the literature in order to put constraints on the growth index γ, especially in the context of the braneworld cosmological model [93]. We would like to stress that in the present work, due to the above caveats, we do not use the $f_{obs}(z)$ data to constrain the cosmological parameters or the growth index. Instead, by using the constraints found from the cosmic expansion data (see Section 3), we shall only compare the evolution of the predicted Liouville growth rate of structure with the data (for a similar analysis see [94]).

In Fig. 3 we display the predicted growth rate Eq. (31) together with the observed $f_{obs}(z)$ [97]. We compare the growth rate of clustering between data and models via a χ^2 statistical test. For the model predictions we use the best fitted values for the parameters (Ω_3 and Ω_δ) obtained in Section 2 for each Liouville model. We then compute the corresponding consistency between models and data ($\chi^2_{min}/11$) and place these results in Table 3. For the Liouville models we find $\chi^2_{min}/11 \simeq 1.19$ while in the case of the concordance ΛCDM cosmology we find $\chi^2_{min}/11 \simeq 0.60$. Note that in the last column of Table 3, we list the corresponding results by excluding from the statistical analysis the observed growth rate of clustering at $z = 3$ [98]. If we consider $\Omega_m = 0.30$, we then find either $\chi^2_{min}/11 \simeq 1.47$ or $\chi^2_{min}(z < 3)/10 \simeq 0.70$.

Finally, let us now examine the case in which the density $\varrho_\delta(z) \propto (1 + z)^\delta$ participates in the clustering. The situation is subtle. As we have discussed above, the exotic matter obeys a modified energy equation, as a result of its direct coupling to the dilaton sources in the conservation Eqs. (6) and (9). Thus, in addition to the existence of pressure in the exotic dark matter fluid, due to a possibly non-trivial equation of state, w_δ, one has the effects of the dilaton coupling that also contribute to the growth equation. For $\phi_0 \sim -1$ in (10), or more generally $\dot{\phi} \sim -H$, the modified

Table 3
Statistical quantification of the comparison between the observed growth rate of clustering (see Table 2) and the theoretical model expectations. The last column corresponds to comparison excluding the observed growth rate of clustering at $z = 3$ [98].

Model	$\chi^2_{min}/11$	$\chi^2_{min}(z < 3)/10$
Q-Data	1.19	0.90
Λ-Data	0.60	0.45

conservation Eq. (11) becomes independent of the equation of state w_δ:

$$\dot{\varrho}_\delta + 3(1+v)H\varrho_\delta \simeq 0, \tag{36}$$

where $v = 1/3$. This energy equation resembles that of a radiation fluid, with effective scaling law for the density $\varrho_\delta \sim a^{-4}$. However, the reader must bear in mind that the energy Eq. (36) stems from the time-dependent dilaton-sources modified energy Eq. (9), which for the case at hand, $\dot{\phi} \sim -H$, is independent of the true equation of state w_δ. Within the standard Cosmological framework, the growth equation for radiation is modified, as compared to that for matter (28), by terms larger by a factor of about $(1+v)(1+3v) = 8/3$ [106]:

$$\ddot{\delta}_\delta + 2H\dot{\delta}_\delta \simeq 4\pi G(1+v)(1+3v)\rho_\delta\delta_\delta. \tag{37}$$

In this case the growing mode scales as $\delta_\delta \sim a^{1+3v}$ at large redshifts which implies that the growth rate takes a large value $f_\delta \sim 1 + 3v = 2$. The latter value seems to be ruled out by the observed growth rate of clustering.

6. Comparison with cluster halo abundances

Since the Liouville cosmology appears not to be ruled out by the growth rate data, it is important to define observational criteria that will enable us to distinguish between it and the concordance ΛCDM cosmology. An obvious choice, that has been extensively used so far, is to extract the theoretical predictions of the models for the cluster-size halo redshift distributions and to confront them with the data. This will allow distinguishing the Q-cosmology from the ΛCDM model. Recently, the halo abundances predicted by a large variety of DE models have been compared with those corresponding to the ΛCDM model [64]. As a result, it is suggested that many DE models (including some of modified gravity) are clearly distinguishable from the ΛCDM cosmology.

We use the Press and Schechter [41] formalism, which determines the fraction of matter that has formed bounded structures as a function of redshift. Mathematical details of such a treatment can be found also in [64]; here we only present the basic ideas. The number density of halos, $n(M,z)$, with masses within the range $(M, M + \delta M)$ is given by:

$$n(M,z)dM = \frac{\bar{\rho}}{M}\frac{d\ln\sigma^{-1}}{dM}f(\sigma)dM, \tag{38}$$

where $\bar{\rho}$ is the mean mass density. In the original Press–Schechter (PSc) formalism, $f(\sigma) = f_{PSc}(\sigma) = \sqrt{2/\pi}(\delta_c/\sigma)\exp(-\delta_c^2/2\sigma^2)$, δ_c is the linearly extrapolated density threshold above which structures collapse [107], while $\sigma^2(M,z)$ is the mass variance of the smoothed linear density field, extrapolated to redshift z at which the halos are identified. It depends on the power-spectrum of density perturbations in Fourier space, $P(k)$, for which we use here the CDM form according to [108], and the values of the baryon density parameter, the spectral slope and Hubble constant according to the recent WMAP7 results [6]. Although the original Press–Schechter mass-function, f_{PSc}, was shown to provide a good first approximation to

that provided by numerical simulations, it was later found to over-predict/under-predict the number of low/high mass halos at the present epoch [109,110]. More recently, a large number of works have provided better fitting functions of $f(\sigma)$, some of them based on a phenomenological approach. In the present treatment, we adopt the one proposed by Reed et al. [42].

We remind the reader that it is customary to parametrize the mass variance in terms of σ_8, the rms mass fluctuations on scales of $8h^{-1}$ Mpc at redshift $z = 0$. In order to compare the mass function predictions of the different cosmological models, it is imperative to use for each model the corresponding value of σ_8 and δ_c. It is well known that for the usual ΛCDM cosmology $\delta_c \simeq 1.675$, while Weinberg & Kamionkowski [53] provide an accurate fitting formula to estimate δ_c for any DE model with a constant equation of state parameter (see their Eq. 18). Although these conditions are not satisfied by our models, one can show [61,64] that the δ_c values for a large family of dark energy models with a time-varying EoS parameter can be well approximated using the previously discussed fitting formula, as long as the EoS parameter remains constant near the present epoch. Since for the current Liouville cosmological model (Q_2 with $\Omega_m = 0.28$) the effective dark energy EoS parameter remains close to $w_{DE} \simeq -0.90$, prior to the present time, implies that we can use the approximate Weinberg and Kamionkowski [53] fitting formula. Doing so we find $\delta_c \simeq 1.668$. As a further consistency check we obtain the δ_c value using the same methodology with that of Pace et al. ([61]; see their Eq. (18)). We find $\delta_c \simeq 1.676$, which is in relatively good agreement with that value found by the fitting formula of [53]. Note that for $\Omega_m = 0.3$ we obtain $\delta_c \simeq 1.674$. Below we utilize those δ_c values which are based on the notation of Pace et al. [61].

Now, in order to estimate the correct normalization of the model's σ_8 power spectrum, we use the formulation presented in Ref. [64] which scales the observationally determined $\sigma_{8,\Lambda}$ value to that of any cosmological model. The corresponding value for the Liouville model is $\sigma_{8,Q_2} = 0.794$ and it is based on $\sigma_{8,\Lambda} = 0.811$ (as indicated also in Table 4), based on the WMAP7 results [6], but also consistent with an average over a variety of recent measurements (see also the corresponding discussion in [64]). For the same model but with $\Omega_m = 0.3$ we obtain $\sigma_{8,Q_2} = 0.847$.

Given the halo mass function from Eq. (38) we can now derive an observable quantity which is the redshift distribution of clusters, $\mathcal{N}(z)$, within some determined mass range, say $M_1 \leqslant M/h^{-1}M_\odot \leqslant M_2 = 10^{16}$. This can be estimated by integrating over mass the expected differential halo mass function, $n(M,z)$:

$$\mathcal{N}(z) = \frac{dV}{dz}\int_{M_1}^{M_2} n(M,z)dM, \tag{39}$$

where dV/dz is the comoving volume element

$$\frac{dV}{dz} = \Omega_s r^2(z)\frac{dr}{dz} \tag{40}$$

and Ω_s is the solid angle. In order to derive observationally relevant cluster redshift distributions and therefore test the possibility of

Table 4
Numerical results: the 1st column indicates the cosmological model. The next three columns list the corresponding Ω_m, σ_8 and δ_c values, respectively. The remaining columns present the fractional relative difference of the abundance of halos between the Liouville and the ΛCDM cosmology for two future cluster surveys discussed in the text and in various redshift intervals. Error bars are 2σ Poisson uncertainties and are shown only if they are larger than 10^{-2}.

Model	Ω_m	σ_8	δ_c	$(\delta\mathcal{N}/\mathcal{N}_\Lambda)_{eROSITA}$		$(\delta\mathcal{N}/\mathcal{N}_\Lambda)_{SPT}$		
				$z < 0.3$	$0.6 \leqslant z < 0.9$	$z < 0.3$	$0.6 \leqslant z < 0.9$	$1.2 \leqslant z < 2$
ΛCDM	0.28	0.811	1.675	0.00	0.00	0.00	0.00	0.00
Liouville Q_2	0.28	0.794	1.676	−0.05	−0.27 ± 0.01	−0.06	−0.15	−0.37
ΛCDM	0.30	0.845	1.675	0.00	0.00	0.00	0.00	0.00
Liouville Q_2	0.30	0.847	1.674	0.00	−0.16 ± 0.01	0.00	−0.02	−0.26

discriminating between the Liouville and the ΛCDM cosmological models, we will use the expectations of two realistic future cluster surveys, extensively used in recent literature:

(a) the eROSITA satellite X-ray survey, with a flux limit of $f_{lim} = 3.3 \times 10^{-14}$ ergs s^{-1} cm^{-2}, at the energy band $0.5 - 5$ keV and covering $\Omega_s \sim 20000$ deg^2 of the sky;

(b) the South Pole Telescope SZ survey, with a limiting flux density at $v_0 = 150$ GHz of $f_{v_0,lim} = 5$ mJy and a sky coverage of $\Omega_s \sim 4000$ deg^2.

To realize the predictions of the first survey we use the relation between halo mass and bolometric X-ray luminosity, as a function of redshift, given in Ref. [111]:

$$L(M,z) = 3.087 \times 10^{44} \left[\frac{ME(z)}{10^{15} h^{-1} M_\odot} \right]^{1.554} h^{-2} \text{ erg s}^{-1}. \quad (41)$$

The limiting halo mass that can be observed at redshift z is then found by inserting in the above equation the limiting luminosity, given by $L = 4\pi d_L^2 f_{lim} c_b$, with d_L the luminosity distance corresponding to the redshift z and c_b the band correction, necessary to convert the bolometric luminosity of Eq. (41) to the $0.5 - 5$ keV band of eROSITA. We estimate this correction by assuming a Raymond–Smith (1977) plasma model with a metallicity of $0.4Z_\odot$, a typical cluster temperature of ~ 4 keV and a Galactic absorption column density of $n_H = 10^{21}$ cm^{-2}.

The predictions of the second survey can be realized using again the relation between limiting flux and halo mass obtained in Ref. [111]:

$$f_{v_0,lim} = \frac{2.592 \times 10^8 \text{ mJy}}{d_A^2(z)} \left(\frac{M}{10^{15} h^{-1} M_\odot} \right)^{1.876} E^{2/3}(z), \quad (42)$$

where $d_A(z) \equiv d_L/(1 + z)^2$ is the angular diameter distance out to redshift z.

Below we shall provide predictions for the Liouville cosmology only for the Q_2 model (see Table 1). For the other Liouville

models we find that our results remain practically the same (due to the small differences in the corresponding Hubble flows). In Fig. 4 (upper panels) we present the expected redshift distributions above a limiting halo mass, which is $M_1 \equiv M_{limit} = $ max$[10^{13.4} h^{-1} M_\odot, M_f]$, with M_f corresponding to the mass related to the flux-limit at the different redshifts, estimated by solving Eq. (41) and Eq. (42) for M. In the lower panels we present the fractional difference between the Liouville model and ΛCDM. The error-bars shown correspond to 2σ Poisson uncertainties, which however do not include cosmic variance and possible observational systematic uncertainties, that would further increase the relevant variance.

The results (see also Table 4) indicate that the redshift variation of the differences between the Q cosmology and ΛCDM model is only slightly affected by variations in the value of Ω_m. For the best-fit case of $\Omega_m = 0.28$ we find that significant model differences should be expected for $z \gtrsim 0.6$, with the Louville model abundance predictions being always less than those of the corresponding ΛCDM model. Therefore, we have verified that there are observational signatures that can be used to differentiate the Liouville model from the ΛCDM and possibly from a large class of DE models (see [64]). In Table 4, one may see a more compact presentation of our results including the relative fractional difference between the Liouville model and the ΛCDM model, in characteristic redshift bins and for both future surveys.

7. Conclusions

In this paper, we have studied the overall dynamics of the Liouville cosmological model in which the dark energy component is associated with the dilaton. In this framework, we first performed a joint likelihood analysis in order to put tight constraints on the main cosmological parameters by using the current observational data (SNIa, BAOs and $H(z)$). We found that the above models can accommodate a late-time accelerated expansion.

Secondly, we performed an additional detailed statistical analysis based on the observed growth rate of clustering and found that the predicted growth rate of various Liouville cosmological models match well the observed growth rate.
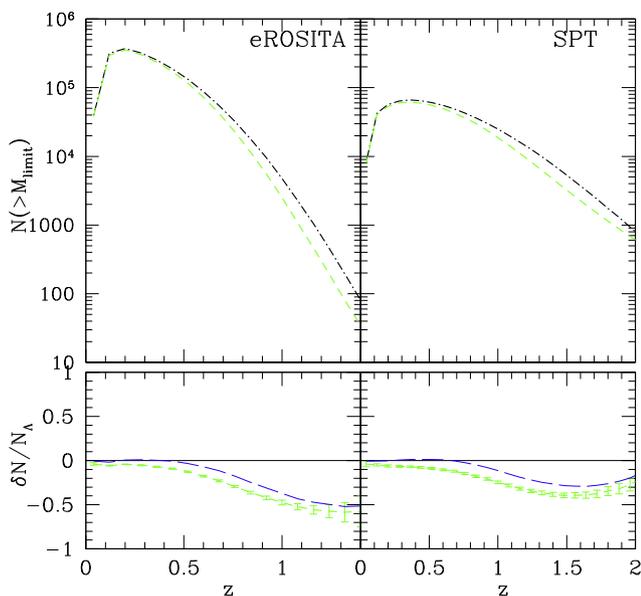
The redshift-dependence of the halo abundances, predicted by the Liouville model, is significantly different from that of the flat ΛCDM cosmology, at relatively high redshifts ($z \gtrsim 0.6$). Future cluster of galaxy surveys can therefore be used to discriminate the Liouville model from other contender DE models.

Acknowledgments

Fig. 4. The expected cluster redshift distribution of the Liouville Q_2 model and ΛCDM (dot-dashed curve) models for $\Omega_m = 0.28$ and for the case of two future cluster surveys (upper panels). The lower panels show the corresponding fractional difference with respect to the reference ΛCDM model (short-dashed curve), while the corresponding curve for the case of $\Omega_m = 0.3$ (see Table 4) is shown as the long-dashed curve. Error bars are 2σ Poisson uncertainties.

Appendix A. Matter density perturbations in dark energy cosmologies

As discussed in the text, the Liouville cosmology, explored in this paper, is close to GR in the matter dominated era (see Eq. (1)

and [66]). Hence, in this appendix we would like to remind the reader, for completeness, of some basic elements of the equations that govern the evolution of the mass density contrast, modelled as an ideal fluid in the framework of non-interacting DE cosmologies that we employ in this work. Also we assume here that for small scales (much smaller than the horizon) the (effective) dark energy component is expected to be smooth and thus we consider perturbations only on the matter component of the cosmic fluid.

In what follows we will adopt a non-relativistic description, based on a continuity equation, together with the Euler and Poisson equations. These equations are:

$$\left(\frac{\partial \rho}{\partial t}\right)_r + \mathbf{\nabla}_r \cdot (\rho \mathbf{u}) = 0 \tag{A1}$$

$$\left(\frac{\partial \mathbf{u}}{\partial t}\right)_r + (\mathbf{u} \cdot \mathbf{\nabla}_r)\mathbf{u} = -\mathbf{\nabla}\Phi, \tag{A2}$$

and

$$\nabla_r^2 \Phi = 4\pi G\rho + 4\pi G\rho_{DE}(3w_{DE} + 1), \tag{A3}$$

where $(\mathbf{r}, t)$ are the proper coordinates, $\mathbf{u}$ is the velocity of a fluid element of volume, ρ is the mass density, ρ_{DE} is the DE density, w_{DE} is the EoS parameter (given in our case by Eq. (27)) and Φ is the gravitational potential. Note that we use here $P_m = 0$ and $P_{DE} = w_{DE}\rho_{DE}$.

Upon changing variables from proper $(\mathbf{r}, t)$ to comoving $(\mathbf{x}, t)$ ones, the fluid velocity becomes

$$\mathbf{u} = \dot{a}\mathbf{x} + a\dot{\mathbf{x}} = \dot{a}\mathbf{x} + \mathbf{v}(\mathbf{x}, t), \tag{A4}$$

while the corresponding differential operators take the following form:

$$\mathbf{\nabla}_x \equiv \mathbf{\nabla} = a\mathbf{\nabla}_r, \tag{A5}$$

and

$$\left(\frac{\partial}{\partial t}\right)_x \equiv \frac{\partial}{\partial t} = \left(\frac{\partial}{\partial t}\right)_r + H\mathbf{x} \cdot \mathbf{\nabla}, \tag{A6}$$

where $\mathbf{x} = \mathbf{r}/a$ and $\mathbf{v}(\mathbf{x}, t)$ is the peculiar velocity with respect to the general expansion. Note that the mass density is written as

$$\rho = \rho_m(t)[1 + \delta_m(\mathbf{x}, t)]. \tag{A7}$$

In this context, using the standard evolution equation for the energy density of ordinary dust,

$$\dot{\rho}_m + 3H\rho_m = 0$$

as well as Eqs. (A5), (A6) and neglecting second-order terms ($\delta_m \ll 1$ and $v \ll u$) we rewrite Eqs. (A1), (A2) and (A3) as:

$$\ddot{a}\mathbf{x} + \frac{\partial \mathbf{v}}{\partial t} + H\mathbf{v} = -\frac{\mathbf{\nabla}\Phi}{a}, \tag{A8}$$

$$\mathbf{\nabla} \cdot \mathbf{v} = -a\frac{\partial \delta_m}{\partial t}, \tag{A9}$$

$$\frac{1}{a^2}\nabla^2 \Phi = 4\pi G\rho_m(1 + \delta_m) + 4\pi G(3w_{DE} + 1)\rho_{DE}. \tag{A10}$$

Following the notation of [88], we can write the gravitational potential as follows

$$\Phi = \phi(\mathbf{x}, t) + \frac{2}{3}\pi G\rho_m a^2 x^2 + \frac{2}{3}\pi G(3w_{DE} + 1)\rho_{DE}a^2 x^2. \tag{A11}$$

Thus, utilizing the Friedmann equation and Eqs. (A11), we arrive, after some algebra, at

$$\frac{\partial \mathbf{v}}{\partial t} + H\mathbf{v} = -\frac{\mathbf{\nabla}\phi}{a}, \tag{A12}$$

and

$$\nabla^2 \phi = 4\pi G a^2 \rho_m \delta_m. \tag{A13}$$

Finally, by taking the divergence of Eq. (A12) and using Eqs. (A10) and (A13), we obtain the time evolution equation for the matter fluctuation field

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0. \tag{A14}$$

In this context, $\delta_m(a) \propto D(a)$, where $D(a)$ is the linear growing mode (usually scaled to unity at the present time). Note that for those cosmological models which adhere to general relativity (GR), the quantity G reduces to the usual Newton's gravitational constant G_N, while in the case of modified gravity models (see [79,82–84]), we have $G = G_{\text{eff}} = Y(a)G_N$ (with $Y(a) \neq 1$).

A well known approximate solution to Eq. (31) is found by [82], where a growth index γ was used to parameterize the linear growing mode for any type of DE with a time varying equation of state. Specifically, the approximated growth factor was defined through

$$D(a) = \exp\left[\int_1^a \frac{\Omega_m^\gamma(u)}{u} du\right], \tag{A15}$$

where now we encapsulate any modification to the G in the term $\Omega_m(a) = \frac{8\pi G}{3H^2} = \frac{\Omega_m a^{-3}}{E^2(a)}$. This completes our discussion on the growth of perturbations due to ordinary matter components of the Liouville cosmology explored in this work.

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